I. True or False:

1. If \( f \) is continuous, then \( \int_1^3 f(v)dv = f(3) - f(1) \).
2. Let \( f(x) \) be integrable, and define \( g(x) = \int_1^x f(x)dx \). Then \( g'(x) = F(x) \), where \( F(x) \) is an antiderivative of \( f(x) \).
3. If \( f \) and \( g \) are continuous and \( f(x) \geq g(x) \) for \( a \leq x \leq b \), then \( \int_a^b f(x)dx \geq \int_a^b g(x)dx \).
4. Let \( F(x) \) be an antiderivative of \( f(x) \). Then \( \int_a^b f(x)dx = \int_a^b F'(x)dx = F(b) - F(a) \).
5. \( \int_0^\frac{\pi}{2} \sin x \cos \frac{x}{2} \, dx = \sin \frac{x}{2} \cos \frac{\pi}{4} \).
6. \( \frac{d}{dx} \int_a^b f(t)dt = f(x) \), where \( a \) and \( b \) are constants.
7. If \( f(-x) = f(x) \), then \( \int_{-a}^a f(x)dx = 0 \).

II. Show your work:

1. All Homework problems.
2. (a) Find the derivatives of the functions:
   
   \( i) \ y = \int_1^x \frac{3}{\sqrt{1 + u}} \, du \)  
   
   \( ii) \ y = \int_{\frac{\pi}{2}}^{\pi} \sqrt{\sin 3x} \, dt \).

   (b) Write the given combinations of integrals as a single integral, and evaluate if possible.

   \( i) \int_0^3 f(x)dx + \int_0^3 f(x)dx - \int_0^3 f(x)dx \)  
   
   \( ii) \int_0^3 f(x)dx + \int_0^3 f(x)dx + \int_0^3 f(x)dx \).

3. Evaluate the integrals. Use substitution whenever necessary. Pay attention to improper integrals.

   \( i) \int \cos^4 x \sin^3 x \, dx \)  
   
   \( ii) \int x^2(1 + 2x^3)^3 \, dx \).

   \( iii) \int \frac{\sec \theta \tan \theta}{\sec \theta} \, d\theta \)  
   
   \( iv) \int \frac{5}{1 + x^2} \, dx \). [Hint: \( \int \frac{1}{1 + x^2} \, dx = \tan^{-1} x + c \).

   \( v) \int (\ln x)^2 \, dx \)  
   
   \( vi) \int e^{-\theta} \cos 2\theta d\theta \).

   \( vii) \int \frac{1}{x^2 \sqrt{x^2 - 9}} \, dx \)  
   
   \( viii) \int \frac{3}{(x-1)(x+1)} \, dx \).

   \( ix) \int^{\infty}_0 \frac{dx}{x^2 + 4x + 3} \)  
   
   \( x) \int^{\infty}_0 \frac{dx}{\sqrt{x^2 + 2x}} \, dx \).

4. Find the area of the region bounded by the curves \( y = x^4 - 4x^2 + 4 \) and \( y = x^2 \).
5. Find the area of the region bounded by the functions \( y^2 - 4x = 4 \) and \( 4x - y = 6 \).
6. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves and the specified axis. Sketch the region and a typical shell.

   \( y = 4x - x^2, \ y = 8x - 2x^2; \ \text{about} \ x = -2 \).

7. Find the volume of the solid obtained by rotating the region bounded by the curves \( x^2 - y^2 = a^2 \) and \( x = a + h \) (where \( a > 0, h > 0 \) about the \( y \)-axis).
8. Find the volume of a solid whose base is a circular disk with radius \( r \), and parallel cross-sections perpendicular to the base are squares.
9. Let \( f(x) = e^x + \frac{1}{4} e^{-x} \) for \( 0 \leq x \leq 1 \). Find the length of the graph of \( f \).
10. A tank has the shape of a hemisphere with radius 5 meters (opening up). It is filled with water to a height of 4 meters. Find the work required to empty the tank by pumping all of the water to the top of the tank. (Water density is \( 1000 \text{ kg/m}^3 \).)
11. Estimate the minimum number of subintervals needed to approximate the integral \( \int_0^3 \frac{1}{\sqrt{x+1}} \, dx \) with an error less than \( 10^{-6} \) by (a) the Trapezoidal Rule and (b) the Simpson’s Rule.
12. Determine if the sequences are convergent: \( a_n = \sin \frac{1}{n} \), and \( a_n = \frac{(-1)^n}{n^2} \).
13. Determine if the series converge. If converges, find its sum. \( \sum_{n=1}^{\infty} \frac{7}{4n} \) and \( \sum_{n=0}^{\infty} \frac{(-1)^n 5}{3n} \), and \( \sum_{n=0}^{\infty} \frac{5^n}{3n + 4} \).