Homework 4

1. Let \( \{V_j, \phi\} \) be an MRA. So \( \phi \) satisfies the scaling equation

\[
\phi(t) = \sqrt{2} \sum_k h(k) \phi(2t - k).
\]

Let \( W_0 \) be the orthogonal complement of \( V_0 \) in \( V_1 \), i.e., \( V_0 \oplus W_0 = V_1 \). Define

\[
\psi(t) = \sqrt{2} \sum_k g(k) \phi(2t - k).
\]

Show that \( \psi \in W_0 \) if and only if

\[
\overline{H(\gamma)}G(\gamma) + \overline{H(\gamma + 1/2)}G(\gamma + 1/2) = 0.
\]

2. One of the choices of \( G \) is that \( g(\gamma) = e^{-2\pi i \gamma \overline{H(\gamma + 1/2)}} \). Verify that

\[
g(k) = (-1)^k \overline{h(1 - k)}.
\]

3. Verify that another choice of \( \{g(k)\} \) can be that

\[
g(k) = (-1)^{N-k} \overline{h(N - k)},
\]

where \( N \) is an odd integer.