Please read the instructions carefully. In problem (1), fill out the table below. Credit will be given only for the correct answers. In all other problems, in order to receive credit, you must show all your work.

The solutions will be posted on the Web (today immediately after the lecture), so will be your grades (Tuesday). The exams will be returned to you during your discussion next week, or during the scheduled office hours of your Teaching Fellow. Finally, before you start, you may wish to consider the words of Albert Einstein (1879–1955): Everything should be made as simple as possible, but not simpler. I would like to wish everybody the best of luck.

Table of answers to questions in problem (1).

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1. This is a multiple choice problem worth 25 points.

(a) Based on the graph of a function $f(x)$ sketched below, choose the only true statement about $f(x)$:

\[
\begin{align*}
\text{(i)} & \quad \lim_{x \to -2} f(x) = 2 \text{ and the graph of } f(x) \text{ has at least one vertical asymptote.} \\
\text{(ii)} & \quad \lim_{x \to -2} f(x) = 1 \text{ and the graph of } f(x) \text{ has at least one horizontal asymptote.} \\
\text{(iii)} & \quad \lim_{x \to -2} f(x) = 1 \text{ and } f(x) \text{ is not continuous at least at one number in its domain.} \\
\text{(iv)} & \quad \lim_{x \to -2} f(x) = 2 \text{ and } \lim_{x \to 1} f(x) = -\infty. \\
\text{(v)} & \quad \text{None of the above.}
\end{align*}
\]

(b) Based on the graph of a function $f(x)$ sketched above, choose the statement about the continuity of $f(x)$ at $x = -2$:

\[
\begin{align*}
\text{(i)} & \quad f(x) \text{ is continuous at } x = -2 \text{ since it is not defined at this point.} \\
\text{(ii)} & \quad f(x) \text{ is continuous at } x = -2 \text{ since in the middle of the gap between two parts of the graph, there is an extra point.} \\
\text{(iii)} & \quad f(x) \text{ is not continuous since the limit } \lim_{x \to -2} f(x) \text{ does not exist.} \\
\text{(iv)} & \quad f(x) \text{ is not continuous since the limit } \lim_{x \to -2} f(x) \text{ does exist, however this limit is not equal to } f(-2). \\
\text{(v)} & \quad \text{None of the above.}
\end{align*}
\]

(c) Let $f(x) = \log(x + 1)$.

\[
\begin{align*}
\text{(i)} & \quad \text{The domain of } f(x) \text{ is the set of all real numbers and the range is the set } \{y : y > -1\}. \\
\text{(ii)} & \quad \text{The domain of } f(x) \text{ is the set } \{x : x > -1\} \text{ and the range is the set of all real numbers.} \\
\text{(iii)} & \quad \text{The domain of } f(x) \text{ is the set of all real numbers and the range is the set of all real numbers.} \\
\text{(iv)} & \quad \text{The domain is the set of all real number such that } x \neq -1 \text{ since } f(x) \text{ has a vertical asymptote } x = -1.
\end{align*}
\]

(d) Relative to the graph of the function $f(x) = \cos x$, the graph of the function $g(x) = 5\cos(x - 3)$ is:

\[
\begin{align*}
\text{A)} & \quad \text{shifted 3 units to the right.} \\
\text{B)} & \quad \text{stretched vertically by a factor of 5.} \\
\text{C)} & \quad \text{shifted to the right by 3 units and compressed vertically by a factor of 5.} \\
\text{D)} & \quad \text{shifted to the right by 3 units and stretched vertically by a factor of 5.} \\
\text{E)} & \quad \text{shifted 5 units upwards and 3 units to the right.}
\end{align*}
\]

(e) Suppose that the graph of a function $g(x)$ is contained only in the first quadrant (that is the set $\{(x,y) : x > 0, y > 0\}$). Then the graph of $g(|x|)$ must lie in:

\[
\begin{align*}
\text{A)} & \quad \text{I quadrant only.} \\
\text{B)} & \quad \text{II quadrant only.} \\
\text{C)} & \quad \text{III and IV quadrants.} \\
\text{D)} & \quad \text{I and II quadrants.} \\
\text{E)} & \quad \text{none of the above.}
\end{align*}
\]
2. (a) Briefly describe (one paragraph) the motion of a car moving along Commonwealth Ave. whose position function \( s(t) \) is shown below \((s(t)\) measures the relative position of the car with respect to the origin). Assume that the Marsh Chapel is the origin and that the positive direction is measured East (towards the Kenmore square). Does the car eventually return to its initial position?

(b) Let \( s(t) \) be the distance function between a female student and her home during a one day period \((t\) is measured in hours). Suppose that student commutes to work without any stops on the her way, then she spends 8 hours at her workplace. Finally, he/she returns home, stopping only once for a few minutes (picking up the groceries). Sketch a possible graph of \( s(t) \).
3. Let

\[ f(x) = \frac{1}{x^2} + \frac{x^2 + 7}{x^2 - 1}. \]

Determine all vertical and horizontal asymptotes of \( f(x) \). Show limit computations in order to receive credit. Horizontal:

\[ \text{Ans.} \]

Vertical:

\[ \text{Ans.} \]
4. Let \( f(x) = \frac{x^3 - 1}{x^2 + 1} \). If \( f(x) \) has an inverse, find it. Otherwise, explain why \( f(x) \) cannot have an inverse function.
5. The displacement (in feet) of a particle moving in a straight line is given by $s = t^2/6$, where $t$ is measured in seconds.

(a) Find the average velocity of the particle during the time intervals: $[1, 3]$, $[1, 2]$ and $[1, 1.5]$.
(b) Find the (instantaneous) velocity when $t = 1$.
(c) What is the graphical interpretation of the velocities in the two previous questions?