\( \Delta_n \quad x_i = \pm x_j \quad 1 \leq i, j \leq n \)

\( \Rightarrow X_{\Delta_n}(q) = \# (x_1, \ldots, x_n) \in \mathbb{F}_q^n : x_i \neq \pm x_j \)

with some
\( n \) choices for which cond s is 0
(as in \( B_n \))
\( (q-1) - (q-2n+1) \) (as in \( B_{n-1} \))

\( X_{\Delta_n}(q) = [(q-1) - (q-2n+1)](q^{2n-2}) + n \)

\( r(\Delta_n) = 2 \cdot 4 \cdots (2n-2) \cdot n = 2^{n-1} \cdot n! \)

\( E_n, E_n, E_n \): meet equlations, but answer oui ou non, too.

Other nice arrangements

Catalan arrangement:

\( C_n : \quad x_i - x_j = -1, 0, 1 \quad 1 \leq i, j \leq n \)

\( C_3 : \)

\( X_{C_n}(q) = \# (x_1, \ldots, x_n) \in \mathbb{F}_q^n \)

such that \( |x_i - x_j| > 1 \)

Choose \( x_1 \rightarrow q \) possibilities

Then "unrump" \( 1 \rightarrow q \):

\( a_1 \quad a_2 \quad a_3 \quad \ldots \quad a_n \)

(\( n \) two consecutive \( q \))

To assign \( x_2, \ldots, x_n \),

- choose \( a_1, a_2, \ldots, a_n \) with \( a_1 + \cdots + a_n = q - n \) \( \binom{q-1}{n-1} \)
- assign \( x_2 \ldots x_n \) the \( n-1 \) \( q \) \( (n-1)! \)

So

\( X_{C_n}(q) = q \binom{n-1}{n-1} = q \binom{q-1}{n-1} \binom{q-2}{n-2} \cdots \binom{q-2n+1}{n-1} \)

and

\( r(C_n) = 1 \cdot 2 \cdot \cdots (n+2)(n+3) \cdots (2n) = (2n)! \cdot \binom{n}{n-1} \)

So in each region of the board arrangement there are \( C_n \) region of \( C_n \). Bijection?

Label each region with the pair \( i,j \) with \( x_i - x_j < 1 \)

Note: \( x_i - x_j < 1 \rightarrow x_i - x_j < 1 \)

Put a dot on each such \( i,j \), so

Then they are in bijection with Dyck paths! So there are \( C_n \) of them