Ex 1: How many ways to pay \( n \) cents with US coins? \((1, 5, 10, 25)\)
\[
f(n) = \# \{(m_1, m_2, m_3, m_4) \in \mathbb{Z}^4 : m_1 + 5m_2 + 10m_3 + 25m_4 = n\}
\]
\[
= \frac{1}{n!} \prod_{p \in \mathbb{P} \cap \mathbb{Z}^d} \left| n \prod_{l \in \mathbb{N} \cap \mathbb{Z}^d} \right|
\]
Note:
\[
\frac{1}{1 - \frac{x}{1 - \frac{x}{1 - \frac{x}{1 - \frac{x}{1 - \frac{x}{1 - \ldots}}}}}} = \sum_{n=0}^{\infty} \frac{x^n}{n!}
\]
So
\[
f(n) = \frac{1}{n!} \left( \frac{A_1}{x_1} + \frac{A_2}{x_2} + \ldots + \frac{A_m}{x_m} \right)
\]
and we can use this to "compute" \( f(n) \).

Now generally:

Let \( A = \{a_1, \ldots, a_m\} \subset \mathbb{Z}^d \)

The partition function
\[
\phi_n(b) = \# \frac{\text{of ways to partition } b \text{ into } a_i^s}{\text{for } n \in \mathbb{N}}
\]
\[
= \# \{ (x_1, \ldots, x_n) \in \mathbb{Z}^d : x_1 + \ldots + x_n = b \}
\]
\[
= \left| P \cap \mathbb{Z}^d \right|
\]
where \( P = \{ x : x \geq 0, Ax = b \} \)

**Notation:** \( be \mathbb{Z}^d \Rightarrow z^b = z_1^{b_1} \ldots z_d^{b_d} \)

**Theorem**
\[
\phi_n(b) = \frac{1}{(1 - z_1)(1 - z_2)(1 - z_3) \ldots (1 - z_n)}
\]

In theory simple.

In practice hard to compute. Lots of interesting theory (algebraic, analytical) and interesting open problems.

Ex 2: Rep th of \( k_n \rightarrow \) partitions parton function

\[
A_H = \{ e_i - e_j : 1 \leq i < j \leq n \} \quad \text{"not system"}
\]
\[
\phi_{A_H}(b) = \# \text{ of } \mathbb{Z}^n \text{ of } k_n:
\]

- \( b = (6, 10, 7, 9) \) (leads at each vertex)

**General facts:**
- previous (analytic) polynomial
- new formula/factorization
- "Chowler renessing"
- "well-graung" formula