where the last equality comes from a given $q-$ analogue identity.

5. (talked with Nina and Emily) Prove that the number of partitions of $n$ in which no part appears exactly once equals the number of partitions of $n$ into parts not congruent to $\pm(\text{mod } 6)$.

We know that the generating function for the number of partitions of $n$ is given by

$$\sum_{n=1}^{\infty} p(n)x^n = \prod_{i=1}^{\infty} (1 + x^i + x^{2i} + \cdots) = \prod_{i=1}^{\infty} \frac{1}{1 - x^i}$$

However, if from one of the sums being multiplied together, we chose $x^i$, (the second element in the sum) this amounts to choosing a partition of $n$ where $i$ only appears once. Therefore the generating function for partitions of $n$ where no element appears exactly once, is

$$\sum_{n=1}^{\infty} q(n)x^n = \prod_{i=1}^{\infty} \frac{1}{1 - x^i} - x^i$$

Now rewriting this last product by breaking up the product of all the $\frac{1}{1 - x^i}$ into a product of all the even numbers $2(\text{ mod } 6)$, $4(\text{ mod } 6)$, and $0(\text{ mod } 6)$, we have

$$\prod_{i=1}^{\infty} \frac{1 + x^{3i}}{1 - x^{2i}} = \left(\prod_{i=1}^{\infty} \frac{1}{1 - x^{6i-4}}\right) \left(\prod_{i=1}^{\infty} \frac{1}{1 - x^{6i-2}}\right) \left(\prod_{i=1}^{\infty} \frac{1}{1 - x^{6i}}\right) \left(\prod_{i=1}^{\infty} \frac{1}{1 + x^{3i}}\right)$$

Now examining the coefficient of $x^n$ we see that it must have taken exponents that were either $2 \text{ mod } 6$, $4 \text{ mod } 6$, $3 \text{ mod } 6$ or $0 \text{ mod } 6$, that is the number of ways to form $x^n$ are
exactly the number of partitions into \( n \) inot parts not parts not congruent to \( \pm( \text{ mod } 6) \), giving us the desired equality.