Another way to represent the compositions of compositions involves dots bars and squiggles.

Draw n dots separated by n−1 spaces.
\[ \bullet \bullet \bullet \bullet = [4] \]

Placing k−1 vertical lines in k−1 of the spaces partitions n (represented by n \( \bullet \)'s) into k parts.
\[ \bullet | \bullet \bullet \bullet = [1+3] \]

Further partition the partition (or rather, further decompose the composition) by placing a squiggle in any of the remaining spaces.
\[ \bullet | \bullet \{ \bullet \bullet \} = [1+(1+2)] \]
\[ \bullet | \bullet \{ \bullet \} \{ \bullet \} = [1+(2+1)] \]
\[ \bullet | \bullet \{ \bullet \} \{ \bullet \} \{ \bullet \} = [1+(1+1+1)] \]

For any n there are n−1 spaces. In any space you may draw a bar, a squiggle or nothing at all. All compositions of compositions of n can be schematically represented in this manner. So the number of compositions of compositions of n is:
\[ 3^{n-1} \]

\( n-1 \) is the number of spaces for each space you have 3 options: 1, \( \$ \) or blank.
\[ \square \]