1. Let $R$ be the ring of continuous functions on $[0, 1]$. Let $I$ be the ideal of function that are zero on some neighborhood of zero,

$$I = \{ f \in R : \exists \epsilon > 0 \text{ with } f = 0 \text{ on } [0, \epsilon] \}.$$ 

Let $\langle f_1, \ldots, f_m \rangle$ be any finite subset of $I$ and let $\epsilon_1, \ldots, \epsilon_m > 0$ so that $f_i = 0$ on $[0, \epsilon_i]$. For every $h \in \langle f_1, \ldots, f_m \rangle$, $h = 0$ on $[0, \min\{\epsilon_1, \ldots, \epsilon_m\}]$. Let $0 < \epsilon < \min\{\epsilon_1, \ldots, \epsilon_m\}$ and find $g \in R$ so that $\{g = 0\} = [0, \epsilon]$. Then $g \in I \setminus \langle f_1, \ldots, f_m \rangle$. Thus $I$ cannot be finitely generated.