

San Francisco State University
Department of Mathematics
Course Syllabus

MATH 471
Introduction to Fourier and Wavelet Analysis

Prerequisites

Grade of C or better in Math 370.

Bulletin Description

Pointwise, uniform, and mean-square convergence of sequences of series and functions; inner product spaces, Fourier series, convergence theorems; an informal treatment of Fourier transforms, inversion, linear filters, Shannon's sampling theorem; Haar wavelets, multi-resolution analysis.

Course Objectives

This is a second course in analysis aimed at introducing students to the mathematics of signal analysis and, at the same time, setting the stage for more advanced topics in real and complex analysis. Students will learn about modes of convergence (pointwise, uniform, and mean-square) of sequences and series of functions in the context of Fourier series, Fourier transforms, and Haar wavelets. It is hoped that this will motivate graduate level topics such as the Lebesgue integral, Hilbert spaces, Banach spaces and complex analysis. Students in the course will also learn to solve problems using concepts of analysis. They will present their solutions as rigorous proofs written in correct mathematical English. They will also be able to use the geometric ideas of inner product spaces to devise computational solutions to problems in signal analysis.

Students who complete this course should be able to:

- Determine the properties of uniformly convergent sequences and series of functions.
- Explain and contrast the different notions of convergence mentioned above.
- Prove and apply the basic convergence theorems for Fourier series of piecewise smooth functions.

- Understand the calculation of Fourier coefficients and sums in the context of orthogonal projection.
- Apply Fourier transform calculations to simple problems in filtering and sampling.
- Understand and implement the concepts of Haar wavelets to decompose and reconstruct a signal.

Evaluation of Students

Students will be graded on their ability to devise, organize and present in correct mathematical English, rigorous solutions to problems. They will also be expected to use computer software to generate solutions and graphs to basic problems in Fourier and wavelet analysis. While instructors may design their own methods of evaluating student performance, these methods must include in-class examinations, graded homework assignments, and a final exam.

Course Outline

Topics	Number of Weeks
Pointwise, uniform, and mean-square convergence of sequences and series of functions.	3
Fourier series of periodic functions: partial sums, the Dirichlet and Fejer kernels, convergence theorems.	3
Fourier integrals: convolution, inversion, Plancherel's formula.	3
Filtering and sampling	1
Haar wavelets	3

Textbooks and Software

A First Course in Wavelets with Fourier Analysis by Albert Boggess and Francis J. Marcowich, Prentice Hall (ISBN 0-13-022809-5).

A First Course in Fourier Analysis by David W. Kammler, Prentice Hall (ISBN 0-13-578782-3).

Submitted by: Eric Hayashi Date: February 24, 2005