San Francisco State University  
Department of Mathematics  
Course Syllabus

MATH 725  
Advanced Linear Algebra

Prerequisites  
Grade of C or better in Math 335 or consent of instructor.

Bulletin Description  
Vector spaces and linear maps on them. Inner product spaces and the finite-dimensional spectral theorem. Eigenvalues, the singular-value decomposition, the characteristic polynomial, and canonical forms.

Course Objectives  
This is a second course in linear algebra in which students make the transition from Euclidean spaces and matrices to abstract vector spaces, inner product spaces and linear transformations. The emphasis is on axiomatic development, proof and conceptual understanding rather than calculation. Students will gain experience working abstractly without coordinates or determinants. In addition, they will learn how ideas from three dimensional geometry can be generalized to unify a wide variety of mathematical applications such as Fourier series, orthogonal functions, and linear regression. This course should pave the way for further study in abstract algebra and advanced analysis. Upon successful completion, students will have a thorough understanding of the proofs of the finite dimensional versions of the Riesz Representation Theorem, the Spectral Theorem for normal operators, polar decomposition, singular value decomposition, and the Jordan canonical form. They will also be able to apply the results to specific operators.

Evaluation of Students  
Instructors will design their own assessment schemes, which usually include weekly graded homework as well as midterm and final exams.
Course Outline

1. Vector spaces, linear independence, bases, dimension. [about 2 weeks]
2. Linear maps, null space, range, matrix representations, invertibility. [2 weeks]
3. Invariant subspaces, upper triangular matrix representations, eigenvectors and eigenvalues. [2.5 weeks]
4. Inner products, norms, orthonormal bases, Gram Schmidt process, orthogonal projection and best approximation, linear functionals and adjoints, Riesz representation. [2.5 weeks]
5. Self-adjoint and normal operators, the Spectral Theorem, positive operators, isometries, polar and singular-value decompositions. [3 weeks]
6. Generalized eigenvalues, characteristic and minimal polynomials, nilpotent operators, Jordan canonical form. [3 weeks]

Textbooks and Software


Submitted by: Eric Hayashi Date: June 3, 2003