1. I’ll hold office hours as scheduled, through 21 May.
2. I presented some material from the unit, Gödel’s and Tarski’s Theorems on Incompleteness and Undefinability.
   a. Its paragraphs 8–10 constitute the logical core of Gödel’s first incompleteness theorem. Gödel’s work [1931] 1967 is considerably more involved, because he presented the theorem in the context of first-order natural-number arithmetic, a theory considerably weaker than the ZF set theory we are using. In our context, the theorem displays a true sentence \( s \) in the formalized ZF such that neither \( s \) nor its negation is provable.
   b. Those paragraphs implicitly rely on our belief that our theorems in informal ZF set theory are true.
   c. Paragraph 12 constitutes the core of Tarski’s undefinability theorem. It shows that there is no formula of formalized ZF that can distinguish true formalized sentences from false ones. Tarski’s work [1936] 1983 was simultaneous with Gödel’s, and used the same methods.
   d. Paragraph 13 shows that the dependence on our belief in ZF can be made more explicit by deriving the results from the hypothesis that formalized ZF be consistent: that no formalized ZF sentence and its negation be both provable.
   e. Paragraph 14 deduces a version of Gödel’s second incompleteness theorem: while the consistency of formalized ZF can be expressed as a formalized ZF sentence, that sentence is not provable, unless ZF is inconsistent (in which case every sentence would be provable).
3. Ms. Yaggie reported about her term project: denotational semantics.
   a. This application of logic and lattice theory provides a framework for deep discussion of the semantics computer languages (and natural languages, for that matter).
   b. She showed that ultrafilters can be used to delineate the concept denoted by a syntactic construct without requiring reference to real objects.
   c. This theory was pioneered by Dana Scott in the 1960s.