1. In class, the Axiom of Choice unit
   a. Reminder: the notation $a_i$ is often used in place of $a(i)$ for the value of a function $a$ corresponding to an argument $i$. For example, a sequence $a$ that you might use in elementary calculus may be a function $a \in \mathbb{R}^\mathbb{N}$, with values $a_0, a_1, \ldots$.
   b. For a function $a \in X^{I \times J}$, we often use notation such as $\bigwedge_j a_{i,j}$ to stand for $\bigwedge \{ a_{i,j} : j \in J \}$. The expression $\bigvee_i \bigwedge_j a_{i,j}$ stands for $\forall i \{ \bigwedge_j a_{i,j} : i \in I \}$.
   c. I’ll discuss assignment 14 briefly.
      i. Routine exercise 3: think small.
      ii. Routine exercise 4: this result suggests an alternative definition of $\preceq$. But if we used that instead of the one adopted in the Cardinals I unit, we would encounter questions much earlier concerning the axiom of choice.
      iii. Routine exercise 5 prepares the way for a method, sometimes used in category theory, to discuss identity functions without referring to the elements of the corresponding sets.
      iv. For most results involving forms of the axiom of choice, you wouldn’t be able to find on your own the functions required until you gain some facility through experience. So on problems such as substantial 5, I give substantial hints. That makes it possible for you to gain some satisfaction from making something work out, while at the same time gaining that experience.
      v. Substantial problem 4 is substantially the same as the step I mentioned earlier, where Prof. Ardila tacitly used the axiom of choice.
   d. Substantial problem 1 shows that any family of functions $f_i$ from a set $X$ to sets $A_i$ can be “simulated” by the projections $c \to c_i$ from the direct product $B$ of the sets $A_i$, and that there is a natural bijection between $B$ and any other set $B'$ that has this property. This allows direct products to be used in category theory, which speaks always of mappings between sets, never of the sets’ elements.
   e. Substantial problem 5 shows how to define the sum and product of an infinite family of cardinals. In parts 2,3 there’s a subtle use of the axiom of choice: from $\forall i [ A_i \sim B_i ]$ conclude $\forall i [ F_i \neq \emptyset ]$, where $F_i$ is the set of all bijections from $A_i \to B_i$; by the axiom of choice there exists $c \in X_i F_i$. Then, for example, you can define a bijection $g : \cup_i A_i \to \cup_i B_i$ by setting $g = \cup_i c_i$. I suspect that the proposition that such a $g$ always exists actually implies the axiom of choice, but I’m not at all sure.