2.3 Axiomatic method

An effective model building technique is the axiomatic method, described by Aristotle around 330 B.C. in the text *Posterior analytics*.\(^1\) He emphasized the need for *precise definition* of the concepts used in a model and its theory, and the *rigorous proof* of theorems about the model. Complicated concepts should be defined, and difficult theorems proved, from simpler ones. Imagine a careful exposition of a mathematical model. Among its concepts are some which are *specific* to the model: they’re not part of the underlying pure mathematics or logic. You can’t define some of these specific concepts, for instance the first one mentioned, in terms of the others without circularity. Thus you must designate certain specific concepts as *undefined* in the model, and define all the others from those. Similarly, among the theorems about the model are some which are specific to it. You can’t deduce some of these specific theorems, for instance the first one mentioned, from the others without circularity. Thus you must designate certain specific theorems as *unproved* in the model, and prove all the others from those. The unproved theorems are called *axioms*.\(^2\) Axioms should be self-evident truths not based on simpler ones. The undefined concepts, axioms and definitions constitute the model. A statement involving the concepts of the model is regarded as a theorem only after it has been rigorously proved from the axioms.

This framework for organizing science was first applied extensively in Euclidean geometry, as described in section 2.4 and practiced in chapter 3. Its *general effectiveness* is clear, however, and Aristotle felt that *all* scholarly inquiries should be reported this way. The method now pervades western culture. One of its most vivid occurrences

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2 Some authors write *postulate* instead of *axiom*, and some use both terms, with slightly different meanings. This book never makes such distinctions, and always uses the word *axiom*. 

outside mathematical disciplines lies in the *Declaration of Independence in Congress, July 4, 1776:*  

**Axioms**  
We hold these truths to be self-evident, that all men are created equal, that they are endowed by their Creator with certain unalienable Rights, that among these are Life, Liberty and the pursuit of Happiness.—That to secure these rights, Governments are instituted among Men, deriving their just powers from the consent of the governed,

**Theorem**  
—That whenever any Form of Government becomes destructive of these ends, it is the Right of the People to alter or to abolish it …

**Axioms**  
(Many destructive acts of the King are enumerated.)

**Theorem**  
The history of the present King of Great Britain is a history of repeated injuries and usurpations, all having in direct object the establishment of an absolute Tyranny over these States.…

**Theorem**  
We, therefore, … declare, That these United Colonies are … Absolved from all Allegiance to the British Crown…

The *Declaration*’s authors, principally Thomas Jefferson, were familiar with the axiomatic method and with Euclid, and assumed that those whom they addressed would be, too.³

The axiomatic method provides general guidelines for organizing a scientific investigation. Once an acceptable axiomatic model has been constructed, the system of undefined concepts, definitions and proofs is useful in reporting the results, and can serve later as an effective framework for learning. Axiomatic models are sometimes adapted for instruction by moving into the list of axioms some theorems with particularly difficult proofs. Students *start* from those theorems instead of struggling with their proofs. On the other hand, polishing an axiom system for archival publication sometimes involves exactly the opposite step. Investigators may discover that some axiom can be proved from the others. The proof may contain interesting and useful arguments. It’s proper, then, to record that by relabeling the axiom as a theorem, and publishing the proof. This also makes the model simpler and perhaps easier to understand.

³ Gary Wills (1978) and I. Bernard Cohen (1995) discuss the wording of the Declaration and Jefferson’s intellectual *milieu* at length. Wills devotes an entire chapter to the use of the term *self-evident*. Cohen’s first two chapters describe the fundamental position of Newtonian science and its Euclidean basis in the education and political thought of that era. Florian Cajori (1890, chapter 1) shows that most college graduates of the time were familiar with Euclid.
Attributing this current description of the axiomatic method entirely to Aristotle is historically questionable. His work is not a polished text, but rough lecture notes, and it’s often obscure. In particular, he may not have seen as clearly as we do now why some particular concepts should be left undefined. Authors using the method sometimes started with circular or nonsensical definitions. Euclid’s are criticized in section 2.4. Jefferson gave no definitions. Until the nineteenth century an axiomatic model was not ordinarily applied to any subject area beyond that for which it was originally devised. To facilitate this practice, which is now common, mathematicians learned to leave the most basic definitions for later specification. Then they realized that those definitions couldn’t be used in proving theorems, hence were superfluous. This observation also eliminated the need for philosophical discussion of the nature of such definitions.

The general status of undefined concepts in axiomatic models—particularly in geometry—remained muddled through the centuries. By the end of the nineteenth, geometric models had been developed that could have been presented entirely axiomatically in the sense just described. But their authors didn’t quite do that, and controversy over the role of undefined concepts continued into the twentieth. As discussed later in section 2.9, the turning point finally occurred around 1900. From then on, the full axiomatic framework was used commonly and routinely.