Midterm Review

Math 228

Fall, 2008

1. True or false:

   (1) For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3, $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$.

   (2) For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3, $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$.

   (3) For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3, $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = \mathbf{0}$.

   (4) For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3, $(\mathbf{u} + \mathbf{v}) \times \mathbf{v} = \mathbf{u} \times \mathbf{v}$.

   (5) The cross product of two unit vectors is a unit vector.

   (6) A linear equation $Ax + By + Cz + D = 0$ represents a line in space.

   (7) The set of points $\{(x, y, z)|x^2 + y^2 = 1\}$ is a circle. (If not, what is it?)

   (8) If $f(x, y) \to L$ as $(x, y) \to (a, b)$ along every straight line through (a, b), then $\lim_{(x,y)\to(a,b)} f(x, y) = L$.

2. Let $\mathbf{u} = \mathbf{i} + \mathbf{j} - \mathbf{2k}$, $\mathbf{v} = \mathbf{3i} - \mathbf{2j} + \mathbf{k}$ and $\mathbf{w} = \mathbf{j} - \mathbf{5k}$. Calculate the following given quantities:

   (1) $6\mathbf{u} - \mathbf{5w}$ (2) $\mathbf{u} \times \mathbf{v}$ (2) $\mathbf{u} \cdot \mathbf{v}$ (4) $|\mathbf{v} \times \mathbf{w}|$ (5) $\mathbf{w} \times \mathbf{w}$.

   (6) Find the projection of $\mathbf{v}$ onto $\mathbf{u}$.

3. Find a unit vector orthogonal to both $< 1, 0, 1 >$ and $< 2, 3, 4 >$.
4. Find the parametric equations for the line that passes through $(-6, -1, 0)$ and $(2, -3, 5)$.
5. Find an equation of the line through $(1, 0, 1)$ and parallel to the line $x = 4t$, $y = 1 - 3t$, $z = 2 + 5t$.
6. Find an equation of the plane that passes through $(-1, 2, 0)$, $(2, 0, 1)$ and $(-5, 3, 1)$.
7. (a) Show that the planes $x + y - z = 1$ and $2x - 3y + 4z = 5$ are neither parallel nor perpendicular.

   (b) find the angle between the above two planes.

8. Find the distance between the two parallel planes $3x + y - 4z = 2$ and $3x + y - 4z = 24$.
9. Find the length of the curve $\mathbf{r}(t) = 2t^{3/2}\mathbf{i} + \cos 2t\mathbf{j} + \sin 2t\mathbf{k}$ for $0 \leq t \leq 1$.
10. Let $\mathbf{r}(t) = t^3/3\mathbf{i} + t^2/2\mathbf{j} + t\mathbf{k}$. Find (a) the unit tangent vector. (b) Set up the integral for the arc length of the section $0 \leq t \leq 2$.
11. A particle starts at the origin with an initial velocity $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$. It s acceleration is $\mathbf{a}(t) = t\mathbf{i} + \mathbf{j} + t^2\mathbf{k}$. Find its position function.
12. Evaluate the limit or show that it does not exist.

    (a) $\lim_{(x,y)\to(0,0)} \dfrac{x^2y^2}{x^2 + 2y^2}$ (b) $\lim_{(x,y)\to(0,0)} \dfrac{2xy}{x^2 + 2y^2}$.

13. Find the first partial derivatives:

    (a) $f(x, y) = \dfrac{x}{\sqrt{x + 2y}}$. (b) $g(u, v, w) = w^2e^{u/v}$.

14. Find all second order partial derivatives:

    (a) $f(x, y, z) = xy^2z^3$. (b) $f(x, y) = x^3 \ln(x - y)$.

15. Find an equation of the tangent plane to the given surface at $(0, 1, 5)$: $z = x^2 + y^2 + 4y$.
16. If $z = \cos xy + y \cos x$, where $x = u^2 + v$ and $y = u - v$, use the Chain Rule to find $\partial z/\partial u$ and $\partial z/\partial v$.
17. Find the directions in which the functions increase and decrease most rapidly at $P_0\,(1, 0)$. Then find the directional derivatives of the function in these directions: $f(x, y) = x^2y + e^{xy} \sin y$.

    Do not overlook the following

18. **Page 543: Chapter 9 Review: 23, 29, 39, 41, 51**
19. **Page 653: Chapter 11 Review: 15, 19, 29, 33, 43, 45**
20. **All homework assignments.**