Review for Final Exam

Math 226

Spring, 2018

Part I Ture or false.

1. $f(x)$ must be defined at a in order for the limit of f to exist at a.
2. If $f(x)$ is continuous at 4 and $f(4) = 2$, then $\lim_{x\to 2} f(4x^2 - 12) = 2$.
3. If $f'(a)$ exists, then $\lim_{x\to a} f(x) = f(a)$.
4. If f is differentiable, then $\dfrac{d}{dx}\sqrt{f(x)} = \dfrac{f^{'}(x)}{2\sqrt{f(x)}}$.
5. If $g(x) = x^5$, then $\displaystyle\lim_{x\to 2} \frac{g(x) - g(2)}{x - 2} = 80$.
6. If $f'(c) = 0$, then f has a local maximum or minimum at c.
7. If f has an absolute minimum value at c, then $f'(c) = 0$.
8. If f is differentiable and $f(-1) = f(1)$, then there is a number c such that $|c| < 1$ and $f'(c) = 0$.
9. If $f'(x) < 0$ for $1 < x < 6$, then $f'(x)$ is decreasing on (1,6).
10. If $f^{''}(2) = 0$, then $(2, f(2))$ is an inflection point of the curve of $y = f(x)$.
11. If $f'(x) = g'(x)$ for $0 < x < 1$, then $f(x) = g(x)$ for $0 < x < 1$.

Part II Show your work.

1. All HW assignments. You are to study all of your HWs!
2. Let $f(x)$ be given by
$$f(x) = \begin{cases} \sqrt{-x}, & x < 0 \\ 3 - x, & 0 \le x < 3 \\ (x-3)^2, & x > 3 \end{cases}$$
   (a) Evaluate each limit, if it exists. (1) $\displaystyle\lim_{x\to 0^-} f(x)$, (2) $\displaystyle\lim_{x\to 0^+} f(x)$, (3) $\displaystyle\lim_{x\to 0} f(x)$, (4) $\displaystyle\lim_{x\to 3^-} f(x)$, (5) $\displaystyle\lim_{x\to 3^+} f(x)$, (6) $\displaystyle\lim_{x\to 3} f(x)$.

   (b) Where is f discontinuous?

   (c) Sketch the graph of f.
3. Evaluate the limit:

   (a) $\displaystyle\lim_{x\to\infty} (\sqrt{x^2+1} - \sqrt{x^2-1})$

   (b) $\displaystyle\lim_{x\to 1} \frac{x^a - 1}{x^b - 1}$ where a, b are constants.

   (c) $\displaystyle\lim_{x\to 1} x^{\frac{1}{x-1}}$. [This is the “logarithmic limit problem”. Let $y = x^{\frac{1}{x-1}}$. Then take the logarithm at both side of the equation, etc.]
4. Find the horizontal and vertical asymptotes of the curve and use them, together with concavity and intervals of increase and decrease, to sketch the curve: $y = \dfrac{1-x}{1+x}$.
5. Find the derivative of the functions:

   (a) $y = \sqrt[5]{x \tan x}$

   (b) $y = \sin(\cos x)$

   (c) $y = 3^{x \sin 3x}$
6. Sketch the graph of a function that satisfies the given conditions: $f(0) = 0$, f is continuous and even, $f'(x) = 2x$ if $0 < x < 1$, $f'(x) = -1$ if $1 < x < 3$, $f'(x) = 1$ if $x > 3$.
7. Find y' using the method of implicit differentiation:
$$y^4 + x^2y^2 + x^4 = y + 1$$

8. Find the absolute maximum value and absolute minimum value of $f = \sqrt{x^2 + 4x + 8}$ on the interval [-3, 0].
9. Let $f(x) = x^4 - 4x^3$.
   (a) Find the intervals on which f is increasing or decreasing.
   (b) Find local maximum and minimum values of f.
   (c) Find the intervals of concavity.
   (d) Find the inflection points.
   (e) Sketch the graph of the functions.
10. [15] The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line. Find (a) the velocity at time t and (b) the displacement traveled during the given time interval.

$$a(t) = 2t + 3, \quad v(0) = -4, \quad 0 \le t \le 3.$$

11. A cylindrical container (with circular base) with an open top is to have a volume of 10π m^3. Material for the base costs \$10 per square meter. Material for the circular side costs \$6 per square meter. Find the dimension of such a container that minimizes the cost of materials.
12. Find the integrals:
   (a) $\displaystyle\int_0^{\pi/4} (x^2 + 1 + \frac{1}{x^2+1})dx.$
   (b) $\displaystyle\int_1^8 \frac{x-1}{\sqrt[3]{x^2}}dx.$
13. (a) Evaluate the integral in terms of area interpretation: $\displaystyle\int_{-3}^{3} \sqrt{9 - x^2}dx.$
   (b) Write the given combination of integrals as a single integral:

$$\int_1^3 f(x)dx + \int_3^6 f(x)dx - \int_{12}^6 f(x)dx$$

14. Find the derivative of the function $y = \displaystyle\int_1^{\cos x} (t + \sin t)dt.$