

# Review for the mid-term exam

Math 376

Fall, 2014

1. Find the solution of the initial value problems:

(1)  $y' = \frac{x}{y}$ ,  $y(0) = 1$ .

(2)  $y' = \frac{y+2x}{x}$ ,  $y(1) = 1$ .

(3)  $\frac{dx}{dt} + x = e^{2t}$ ,  $x(0) = 1$ .

(4)  $y' + y \tan x = \sin 2x$  on  $(-\frac{\pi}{2}, \frac{\pi}{2})$ , with  $y = 2$  when  $x = 0$ .

2. In each case, find a second order linear differential equation satisfied by  $u_1$  and  $u_2$ .

(1)  $u_1(t) = e^t$ ,  $u_2(t) = e^{-t}$ .

(2)  $u_1(t) = e^{2t}$ ,  $u_2(t) = te^{2t}$ .

(3)  $u_1(t) = e^{-t/2} \cos t$ ,  $u_2(t) = e^{-t/2} \sin t$ .

3. (a) Solve the differential equation  $(x + \sin y) + (x \cos y - 2y) \frac{dy}{dx} = 0$ .

(b) Consider the solution curve satisfying  $y(10) = 4$ . Find an expression for the constant of integration in this case.

4. Two 100-gallon tanks are connected so that the overflow of the first enters the second. A salt solution with a concentration of 2 pounds per gallon enters the first tank at the rate of 2 gallons per minute, and the resulting solution overflows into the second tank at the same rate. Meanwhile, pure water also enters the second tank from another source at the rate of 3 gallons per minute, with a resulting overflow of 5 gallons per minutes.

(a) If the first tank initially contains pure water, and the second contains 10lb of salt in solution, find the amount of salt in each tank at time  $t \geq 0$ .

(b) What happens to the amount of salt in each tank as  $t \rightarrow \infty$ ? Explain.

(c) Do the two tanks ever contain an equal amount of salt? Explain.

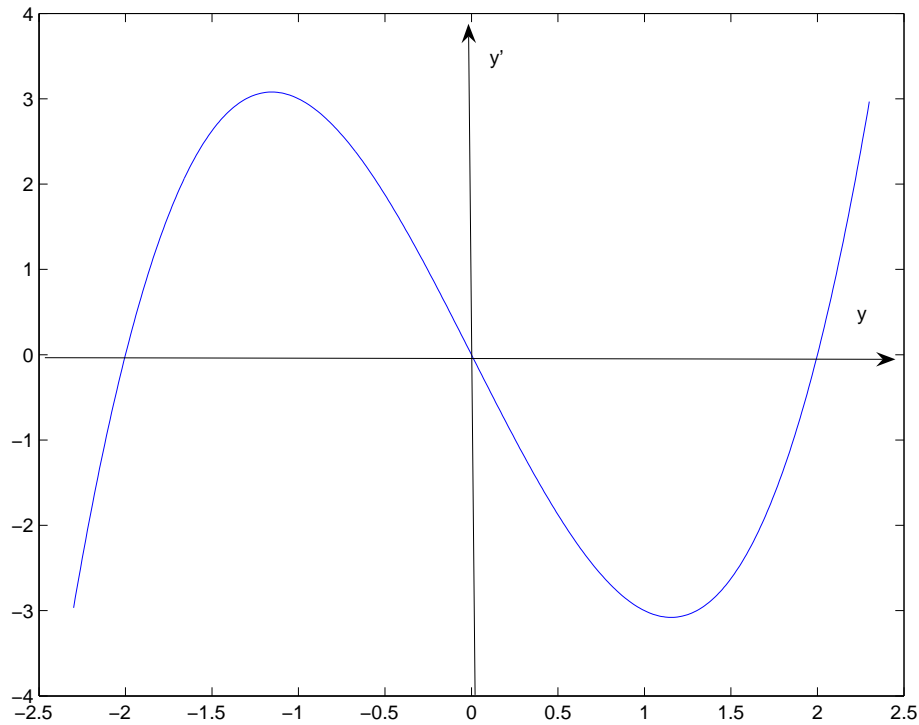
5. A body of mass  $m$  with initial velocity  $v_0$  is subject to a drag resistance proportional (with a constant  $k$ ) to the magnitude of velocity.

(a) Show that the velocity at time  $t$  is

$$v(t) = \frac{mg}{k} + (v_0 - \frac{mg}{k})e^{-\frac{kt}{m}}.$$

(b) Find the limit as  $k$  tend to 0 of  $v(t)$ . Does this agree with the free fall formula  $v_0 + gt$ ? Explain.

6. Consider the following graph of the function  $f(x)$ . Based on the graph, draw the solution curves  $x(t)$  to the differential equation  $\frac{dx}{dt} = f(x)$  for  $t \geq 0$ . Indicate the equilibrium solutions and discuss their stability.



7. Solve the differential equation:  $y'' + 2y' + 2y = 0$  given  $y(\pi/4) = 2$  and  $y'(\pi/4) = -2$ .
8. Use the method of reduction of order to find a second solution of the given differential equation

$$xy'' - y' + 4x^3y = 0, \quad x > 0, \quad y_1(x) = \sin(x^2).$$

9. Find the general solution of the differential equation

$$2y'' + 3y' + y = t^2 + 3 \sin t.$$