Review 2

Math 325

Fall, 2011

Part I. All HW problems of covered sections of Chapters 3,5,6,7. Note again that Chapter 3 is included in the final. Review problems of Chapter 3 on Review 1.

Part II. Ture or False (Some of these problems require calculations)

1. The equation $(AB)^T = A^T B^T$ for all $n \times n$ matrices A and B.
2. All nonzero symmetric matrices are invertible.
3. If $\vec{u}$ is a unit vector in R^n, and $L = span(\vec{u})$, then $proj_L(\vec{x}) = (\vec{x} \cdot \vec{u})\vec{x}$ for all $\vec{x}$ in R^n.
4. If A and B are symmetric $n \times n$ matrices, then $ABBA$ must be symmetric as well.
5. There exists a subspace V of R^5 such that $dim(V) = dim(V^\perp)$, where $V^\perp$ is the orthogonal complement of V.
6. If $\vec{x}$ and $\vec{y}$ are two vectors in R^n, then the equation $\|\vec{x} + \vec{y}\|^2 = \|\vec{x}\|^2 + \|\vec{y}\|^2$ must hold.
7. If A is any matrix with $Ker(A) = \{\vec{0}\}$, then the matrix AA^T represents the orthogonal projection onto the range of A.
8. If V is a subspace of R^n and $\vec{x}$ is a vector in R^n, then vector $proj_V\vec{x}$ must be orthogonal to vector $\vec{x} - proj_V\vec{x}$.
9. The formula $ker(A) = ker(A^T A)$ holds for all matrices A.
10. If the entries of two vectors $\vec{v}$ and $\vec{w}$ in R^n are all positive, then $\vec{v}$ and $\vec{w}$ must enclose an acute angle.
11. The formula $(ker(B))^\perp = im(B^T)$ holds for all matrices B.
12. If $AA^T = A^2$ for a 2 x 2 matrix A, then A must be symmetric.
13. If A is any symmetric 2 x 2 matrix, the there must exist a real number x such that the matrix $A - xI_2$ fails to be invertible.
14. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, then the matrix Q in the QR-factorization of A is a rotation matrix.
15. The eigenvalues of any triangular matrix are its diagonal entries.
16. The algebraic multiplicity of an eigenvalue cannot exceed its geometric multiplicity.
17. If an $n \times n$ matrix A is diagonalizable (over R), then there must be a basis of R^n consisting of eigenvectors of A.
18. If the standard vectors $\vec{e}_1, \cdots, \vec{e}_n$ are eignevectors of an $n \times n$ matrix A, the A must be diagonal.
19. If $\vec{v}$ is an eigenvector of A, then $\vec{v}$ must be an eigenvector of A^3 as well.

20. If 0 is n eigenvalue of a matrix A, then $det(A) = 0$.

21. If 1 is the only eigenvalue of an $n \times n$ matrix A, then A must be I_n.

22. If A and B are $n \times n$ matrices, if α is an eigenvalue of A, and if β is an eigenvalue of B, then $\alpha\beta$ must be an eigenvalue of AB.

23. If 3 is an eigenvalue of A, then 9 must be an eigenvalue of A^2.

24. If two $n \times n$ matrices A and B are diagonalizable, the AB must be diagonalizable as well.

25. If an invertible matrix A is diagonalizable, then A^{-1} must be diagonalizable as well.

26. All invertible matrices are diagonalizable.

27. All diagonalizable matrices are invertible.

28. All symmetric 2 x 2 matrices are diagonalizable (over $\mathbf{R}$)

29. A 2 x 2 reflection matrix about a line L must be diagonalizable.

30. If vector $\vec{v}$ is an eigenvector of both A and B, then $\vec{v}$ must be an eigenvector of $A + B$.

31. If $\vec{v}$ and $\vec{w}$ are linearly independent eigenvectors of matrix A, then $\vec{v} + \vec{w}$ must be an eigenvector of A as well. If false, when would this be true?

32. If an $n \times n$ matrix A is diagonalizable, then A must have n distict eigenvalues.

33. If a matrix is diagonalizable, then the algebraic multiplicity of each eigenvalue λ must equal the geometric multiplicity of λ.

34. If an $n \times n$ matrix A is diagonalizable (over R), then every vector $\vec{v}$ in R^n can be expressed as a sum of eigenvectors of A.

35. If A is a 2×2 matrix with eigenvalues 3 and 4, and if $\vec{u}$ is a unit eigenvector of A, then the length of the vector $A\vec{u}$ cannot exceed 4.