Review for the mid-term exam

Math 325

Fall, 2011

Part I. All HW problems of Chapters 1 - 3, 5.1, 5.2.
Part II. Ture or False (Some of these problems require calculations)

1. A system of 4 linear equations in three unknowns is always inconsistent.
2. If the 4x4 matrix A has rank 4, then any linear system with the coefficient matrix A will have a unique solution.
3. There exists a 5x5 matrix A of rank 4 such that the system $A\vec{x} = \vec{0}$ has only the solution $\vec{x} = \vec{0}$.
4. Vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is a linear combination of vectors
$$\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \text{ and } \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$
5. If A is a nonzero matrix of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then the rank of A must be 2.
6. There exists a 2x2 matrix A such that
$$A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ and } A \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$
7. If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are nonzero vectors in R^2, then $\vec{w}$ must be a linear combination of $\vec{u}$ and $\vec{v}$.
8. If $\vec{u}$ is a linear combination of vectors $\vec{v}$ and $\vec{w}$, and $\vec{v}$ is a linear combination of vectors $\vec{p}$, $\vec{q}$, and $\vec{r}$, then $\vec{u}$ must be a linear combination of $\vec{p}$, $\vec{q}$, and $\vec{r}$ and $\vec{w}$.
9. If A is a 4x3 matrix of rank 3 and $A\vec{v} = A\vec{w}$ for two vectors $\vec{v}$ and $\vec{w}$ in R^3, then $\vec{v} = \vec{w}$.
10. The function $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x - y \\ y - x \end{bmatrix}$ is a linear transformation.
11. Matrix $\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix}$ represents a rotation.
12. If $AB = I_n$ for two $n \times n$ matrices A and B, the A must be the inverse of B.
13. The function $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ 1 \end{bmatrix}$ is a linear transformation.
14. Matrix $\begin{bmatrix} -0.68 & 0.8 \\ -0.8 & -0.6 \end{bmatrix}$ represents a rotation.
15. $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}^3 = \begin{bmatrix} 1 & 3k \\ 0 & 1 \end{bmatrix}$ for all real numbers k.
16. If $A^2 = I_n$, then A (matrix) must be invertible.

17. The equation $A^{-1} = A$ holds for all 2x2 matrices A representing a reflection.
18. There exist a 2x3 matrix A and a 3x2 matrix B such that $AB = I_2$.
19. There exist a 3x2 matrix A and a 2x3 matrix B such that $AB = I_3$.
20. If the linear system $A^2\vec{x} = \vec{b}$ is consistent, then the system $A\vec{x} = \vec{b}$ is consistent.
21. If $\vec{v}_1$, $\vec{v}_2$, ..., $\vec{v}_n$ are linearly independent vectors in R^n, then they must form a basis of R^n.
22. The kernel of any invertible matrix consists of the zero vector only.
23. If $2\vec{u} + 3\vec{v} + 4\vec{w} = 5\vec{u} + 6\vec{v} + 7\vec{w}$, then vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ must be linearly dependent.
24. The column vectors of a 5x4 matrix must be linear dependent.
25. If the kernel of a matrix A consists of the zero vector only, then the column vectors of A must be linearly independent.
26. If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are in a subspace V of R^n, then the vector $2\vec{u} + 3\vec{v} + 4\vec{w}$ must be in V as well.
27. If vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ are linearly independent, then vectors $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$ are linearly independent.
28. Matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is similar to $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.
29. If a 2x2 matrix P represents the orthogonal projection onto a line in R^2, then P must be similar to the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.
30. If vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ are linearly dependent, then vector $\vec{w}$ must be a linear combination of $\vec{u}$, and $\vec{v}$.
31. If A and B are invertible matrices of size $n \times n$, then AB must be similar to BA.
32. If $AB = 0$ for two 2x2 matrices A and B, then BA must be the zero matrix as well.
33. R^2 is a subspace R^3.
34. For every subspace V of R^3, there exists a 3x3 matrix A such that $V = Im(A)$.
35. If the kernel of a 5x4 amtrix A consists of the zero vector alone, and if $AB = AC$ for two 4x5 matrix B and C, the matrices B and C must be equal.
36. There is a subspace V of R^5 such that dim(V)=dim($V^\perp$), where $V^\perp$ denotes the orthogonal complement of V.
37. Every invertible $n \times n$ matrix A can be expressed as the product of an orthogonal matrix (columns of the matrix forms an ONB of R^n) and an upper triangular matrix.
38. If $\vec{x}$ and $\vec{y}$ are two vectors in R^n, then the equation $\|\vec{x} + \vec{y}\|^2 = \|x\|^2 + \|y\|^2$ must hold.
39. Let $\vec{v}_1, \cdots, \vec{v}_m$ be a basis of V. A vector $\vec{x}$ in R^n is orthogonal to V if and if $\vec{x}$ is orthogonal to all the vectors $\vec{v}_1, \cdots, \vec{v}_m$.
40. Calculate: find the length of the vector $\vec{x} = 7\vec{u}_1 + \vec{u}_2 - 4\vec{u}_3 - 2\vec{u}_4 + \vec{u}_5$, where $\vec{u}_1, \cdots, \vec{u}_5$ are orthonormal vectors in R^{10}.
41. Among all vectors of R^n whose components add up to 1, find the vector of minimal length.
42. Can you find a line L in R^n and a vector $\vec{x}$ in R^n such that $\vec{x} \cdot Proj_L\vec{x} < 0$? Why?
43. If A has orthogonal columns, but not orthonormal. What are the Q and R in the QR-factorization?