I. True or False:

1. If $f$ is continuous, then $\int_0^3 f(v) dv = f(3) - f(1)$.
2. Let $f(x)$ be integrable, and define $g(x) = \int_x^1 f(x) dx$. Then $g'(x) = F(x)$, where $F(x)$ is an antiderivative of $f(x)$.
3. If $f$ and $g$ are continuous and $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x) dx \geq \int_a^b g(x) dx$.
4. Let $F(x)$ be an antiderivative of $f(x)$. Then $\int_a^b f(x) dx = \int_a^b F'(x) dx = F(b) - F(a)$.
5. $\int_0^\pi \frac{d}{dx} (\sin^2 x) dx = \sin^2 x + \frac{1}{2} x |^\pi_0$.
6. $\int_a^b f(t) dt = f(x)$, where $a$ and $b$ are constants.
7. If $f(-x) = f(x)$, then $\int_{-a}^a f(x) dx = 0$.

II. Show your work:

1. All Homework problems.
2. (a) Find the derivatives of the functions:
   (i) $y = \int_0^3 \frac{\sin x}{\sqrt{1+x^2}} dx$ (ii) $y = \int_b^x \sqrt{\sin t} dt$.
   (b) Write the given combinations of integrals as a single integral, and evaluate if possible.
   (i) $\int_{-3}^3 f(x) dx + \int_0^6 f(x) dx - \int_0^3 f(x) dx$ (ii) $\int_{-3}^3 f(x) dx + \int_3^4 f(x) dx + \int_4^6 f(x) dx$.
3. Evaluate the integrals. Use substitution whenever necessary. Pay attention to improper integrals.
   (i) $\int \cos^4 x \sin^3 x dx$ (ii) $\int x^2(1 + 2x^3)^3 dx$.
   (iii) $\int \frac{\sec \theta \tan \theta}{1 + \sec \theta} d\theta$ (iv) $\int \frac{x}{1+x^4} dx$. [Hint: $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$].
   (v) $\int (\ln x)^2 dx$ (vi) $\int e^{-\theta} \cos 2\theta d\theta$.
   (vii) $\int \frac{1}{x^2 \sqrt{x^2 - 9}} dx$ (viii) $\int \frac{3x^2 - 4x + 5}{(x-1)(3x+1)} dx$.
   (ix) $\int \frac{dx}{x^2 + 4x + 5}$ (x) $\int_{-\infty}^{\infty} \frac{1}{x^2 + 2} dx$.
4. A particle moves along a line with the velocity function $v(t) = t^2 - t$. Find (a) the displacement and (b) the distance traveled by the particular during the time interval $[0, 5]$.
5. Find the area of the region bounded by the curves $y = x^4 - 4x^2 + 3$ and $y = x^2$.
6. Find the area of the region bounded by the functions $y^2 = 4x - 4$ and $4x - y = 6$.
7. Use the method of cylindrical shell to find the volume generated by rotating the region bounded by the given curves and the specified axis. Sketch the region and a typical shell.
   $y = 4x - x^2$, $y = 8x - 2x^2$; about $x = -2$.
8. Find the volume of the solid obtained by rotating the region bounded by the curves $x^2 + y^2 = a^2$ and $x = a + h$ (where $a > 0, h > 0$) about the $y$-axis.
9. Find the volume of a solid whose base is a circular disk with radius $r$, and parallel cross-sections perpendicular to the base are squares.
10. Let $f(x) = e^x + \frac{1}{2} e^{-x}$ for $0 \leq x \leq 1$. Find the length of the graph of $f$.
11. A tank has the shape of a hemisphere with radius 5 meters (opening up). It is filled with water to a height of 4 meters. Find the work required to empty the tank by pumping all of the water to the top of the tank. (Water density is 1000 kg/m$^3$.)
12. Estimate the minimum number of subintervals needed to approximate the integral $\int_0^3 \frac{1}{\sqrt{x+1}} dx$ with an error less than $10^{-6}$ by (a) the Trapezoidal Rule and (b) by the Simpson’s Rule.