Mathematics 227

Review for the mid-term exam
Fall 2015

I. True or False:

1. If f is continuous, then $\int_1^3 f(v)dv = f(3) - f(1)$.
2. Let $f(x)$ be integrable, and define $g(x) = \int_1^x f(x)dx$. Then $g'(x) = F(x)$, where $F(x)$ is an antiderivative of $f(x)$.
3. If f and g are continuous and $f(x) \geq g(x)$ for $a \leq x \leq b$, then $\int_a^b f(x)dx \geq \int_a^b g(x)dx$.
4. Let $F(x)$ be an antiderivative of $f(x)$. Then $\int_a^b f(x)dx = \int_a^b F'(x)dx = F(b) - F(a)$.
5. $\int_0^{\frac{\pi}{2}} \frac{d}{dx}(\sin\frac{x}{2}\cos\frac{x}{3})dx = \sin\frac{x}{2}\cos\frac{x}{3}\Big]_0^{\frac{\pi}{2}}$.
6. $\frac{d}{dx}\int_a^b f(t)dt = f(x)$, where a and b are constants.
7. If $f(-x) = f(x)$, then $\int_{-a}^a f(x)dx = 0$.
8. $\int_0^4 \frac{x}{x^2-1}dx = \frac{1}{2}\ln 5$.
9. $\int_1^\infty \frac{1}{x^{\sqrt{2}}}dx$ is convergent.

II. Show your work:

1. All Homework problems.
2. (a) Find the derivatives of the functions:

   (i) $y = \int_0^{x^3} \frac{u}{\sqrt{1+u^3}}du$ (ii) $y = \int_{\sqrt{x}}^{x^3} \sqrt{t}\sin t dt$.

   (b) Write the given combinations of integrals as a single integral, and evaluate if possible.

   (i) $\int_{-3}^5 f(x)dx + \int_5^6 f(x)dx - \int_{-3}^0 f(x)dx$. (ii) $\int_3^4 f(x)dx + \int_1^3 f(x)dx + \int_4^1 f(x)dx$.
3. Evaluate the integrals. Use substitution whenever necessary. Pay attention to improper integrals.

   (i) $\int \cos^4 x \sin^3 x dx$. (ii) $\int x^2(1+2x^3)^3 dx$.

   (iii) $\int \frac{\sec\theta\tan\theta}{1+\sec\theta}d\theta$. (iv) $\int \frac{x}{1+x^4}dx$. [Hint: $\int \frac{1}{1+x^2}dx = \tan^{-1}x + c$].

   (v) $\int(\ln x)^2 dx$ (vi) $\int e^{-\theta}\cos 2\theta d\theta$

   (vii) $\int \frac{1}{x^2\sqrt{x^2-9}}dx$ (viii) $\int \frac{3x^2-4x+5}{(x-1)(x^2+1)}dx$

   (ix) $\int_0^3 \frac{dx}{x^2-6x+5}$ (x) $\int_{-\infty}^\infty \frac{x}{1+x^2}dx$
4. Find the area of the region bounded by the curves $y = x^4 - 4x^2 + 4$ and $y = x^2$.
5. Find the area of the region bounded by the functions $y^2 - 4x = 4$ and $4x - y = 6$.
6. Use the method of cylindrical shell to find the volume generated by rotating the region bounded by the given curves and the specified axis. Sketch the region and a typical shell.

$$y = 4x - x^2, \quad y = 8x - 2x^2; \quad \text{about } x = -2.$$

7. Find the volume of the solid obtained by rotating the region bounded by the curves $x^2 - y^2 = a^2$ and $x = a + h$ (where $a > 0$, $h > 0$) about the y-axis.
8. Find the volume of a solid whose base is a circular disk with radius r, and parallel cross-sections perpendicular to the base are squares.
9. Let $f(x) = e^x + \frac{1}{4}e^{-x}$ for $0 \leq x \leq 1$. Find the length of the graph of f.
10. A tank has the shape of a hemisphere with radius 5 meters (opening up). It is filled with water to a height of 4 meters. Find the work required to empty the tank by pumping all of the water to the top of the tank. (Water density is 1000 kg/m^2.)
11. Estimate the minimum number of subintervals needed to approximate the integral $\int_0^3 \frac{1}{\sqrt{x+1}}dx$ with an error less than 10^{-6} by (a) the Trapezoidal Rule and (b) by the Simpsons Rule.