Mathematics 227

Review for the Final Exam

Fall, 2015

Part I. Determine whether the statement is true or false:

1. If $\lim_{n\to\infty} a_n = 0$, then $\sum_n a_n$ is convergent.
2. If $\sum c_n 6^n$ is convergent, then $\sum c_n(-2)^n$ is convergent.
3. If $\sum c_n 6^n$ is convergent, then $\sum c_n(-6)^n$ is convergent.
4. If $\sum c_n x^n$ diverges at $x = 6$, then it diverges when $x = 10$.
5. If $0 \leq a_n \leq b_n$, and $\sum b_n$ diverges, then $\sum a_n$ diverges.
6. If $\sum a_n$ is divergent, then $\sum |a_n|$ is divergent.
7. If $f(x) = 2x - x^2 + \frac{1}{3}x^3 - \cdots$ converges for all x, then $f^{'''}(0) = 2$.
8. If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.
9. If $a_n > 0$ and $\sum a_n$ converges, then $\sum(-1)^n a_n$ converges.
10. If $p > 1$, then $\sum_{k=1}^{\infty} \frac{8}{k^p}$ converges.

Part II. Show your work:

1. **All homework assignments**.
2. Evaluate $\displaystyle\int_0^{\ln 3} e^x\sqrt{1+e^x}\ \text{dx}$
3. Evaluate $\displaystyle\int \frac{10}{(x+1)(x^2+4)}\ \text{dx}$
4. Find the average of the function $f(x) = x^2\cos x$ on the inteval $[0, \pi]$.
5. Find the area of the region bounded by $x + y = 0$ and $x = y^2 + 3y$.
6. Find the volume of the solid obtained by rotating the region bounded by the curves $x = 2y - y^2$ and $x = 0$ about the y-axis.
7. Let $f(x) = e^x + \dfrac{1}{4}e^{-x}$ for $0 \leq x \leq 1$. Find the arc-length of the graph of f.
8. Find the volume of a solid whose base is a circular disk with radius r, and parallel cross-sections perpendicular to the base are squares.
9. A tank has the shape of a hemisphere with radius 5 meters. It is filled with water to a height of 4 meters. Find the work required to empty the tank by pumping all of the water to the top of the tank. (Water density is 1000 kg/m^2.)
10. Sketch the curve of the equation $r = \cos 2\theta$ in polar coordinate.
11. Find the area of the region outside of $r = 1$ and inside of $r = 4\cos\theta$.
12. Find the Cartesian (xy-) equation of the curves: (a) $r\sin\theta = 2$, and (b) $r = 2\sin\theta$.
13. Find the polar equation of $2xy = 1$.

14. Determine if the integrals are convergent. Evaluate those that are convergent.

$$\int_2^\infty \frac{1}{(x-3)^{3/2}}dx, \qquad \int_0^2 \frac{x-3}{2x-3}dx$$

15. Use the Simpson's Rule with $n = 6$ subintervals to approximate $\int_1^4 (\ln x)^2 \text{ dx}$. Determine the error bound of this approximation.

16. Determine if the series is convergent. If it is convergent, find the sum.

$$\sum_{k=0}^{\infty} 4^{-k}, \qquad \sum_{k=0}^{\infty} \frac{2k}{k+3}$$

17. Determine if the series converges absolutely, conditionally or diverges. $\sum_{k=1}^{\infty}(-1)^k \frac{k}{k^2+1}$

18. Find the radius and interval of convergence $\sum_{k=0}^{\infty} \frac{4}{k!}(x-2)^k$.

19. Find the Taylor polynomial of degree 4 of $f(x) = \sin x + e^{3x}$ about the center $a = 0$.

20. Find the Taylor series of $f(x) = \ln(1+x)$ about the center $a = 0$.