Homework 7

Math 471

1. Let $\{V_j, \phi\}$ be an MRA. $\phi \in V_0$ is the scaling function. Show that a function $\psi \perp V_0$ if and only if
$$\sum_{k \in \mathbf{Z}} \hat{\phi}(\gamma + k)\overline{\hat{\psi}(\gamma + k)} = 0 \qquad (a.e.)$$
(Note, there will be a switch of limit in the proof. Make sure it is addressed.)

2. (Continuation of the previous problem). Suppose W_0 is the orthogonal complement of V_0 in V_1. How would you ensure that $\psi \in W_0$?

3. Let $\{h(n)\}$ and $\{g(n)\}$ be the scaling and wavelet sequences, respectively. Show that
$$\phi_{1k} = \sum_n \overline{h(k-2n)}\phi_{0n} + \sum_n \overline{g(k-2n)}\psi_{0n}$$
if and only if
$$\begin{cases} |H(\gamma)|^2 + |G(\gamma)|^2 & = & 2 \\ H(\gamma)\overline{H(\gamma + \frac{1}{2})} + G(\gamma)\overline{G(\gamma + \frac{1}{2}} & = & 0 \end{cases}$$
[Hint: Consider $k = 2m$ and $k = 2m + 1$ separately. You'll also need to figure out the DFT of $\{h(2m)\}$ and $\{h(2m+1)\}$ in terms of $H(\gamma)$, etc.]

4. State the definition of the system matrix $A(\gamma)$. Suppose that $H(\gamma)$ generates an MRA (so that $H(\gamma)$ satisfies needed condition for the MRA). Verify that the choice of $G(\gamma) = e^{-2\pi i\gamma}\overline{H(\gamma + \frac{1}{2})}$ ensures that $A(\gamma)$ is a unitary matrix. Show also
$$g(n) = (-1)^n\overline{h(1-n)}.$$

5. We derived Daubechies orthonormal wavelets in two different ways in class. Summarize the principle and/or steps of the construction. Use a small example in needed, but not just examples without principle and step explanations.