1. Exercise 3.31, [DW], page 75.

2. Prove Theorem 3.42, [DW], page 80, parts (c), (d), (e).

3. Prove Theorem 3.43, [DW], page 80.

4. Prove the following theorem. Let \( \phi \in L^2 \), and \( V_0 \equiv \{ T_n \phi \} \). Define
\[
V_j \equiv \left\{ f \in L^2 \left| f(2^{-j}t) \in V_0 \right. \right\}.
\]

Then if \( \{ T_n \phi \} \) is an ONB of \( V_0 \), then \( \{ D_j T_n \phi \} \) is an ONB of \( V_j \). Here \( T_n f \equiv f(\cdot - n) \), and \( D_j f = 2^{j/2} f(2^j \cdot) \).

5. [Optional] Let \( \phi \in L^2(\mathbb{R}) \) be such that \( \hat{\phi}(\gamma) = 1 \), for \( |\gamma| \leq \Omega/2 \), and \( \hat{\phi} \) decays to zeros continuously for \( |\gamma| > \Omega/2 \). Derive a sampling formula of (or similar to) the following form:
\[
\forall f \in PW_{\Omega/2}, \quad f(t) = \sum_{n \in \mathbb{Z}} f\left(\frac{n}{\Omega}\right)\phi(t - \frac{n}{\Omega}),
\]
where \( PW_{\Omega/2} \) is the Paley Winer (band-limited) subspaces of functions \( f \in L^2 \) such that \( \text{supp}(\hat{f}) \subseteq [-\Omega/2, \Omega/2] \). [Hint: work from the identity \( \hat{f} = \hat{f} \hat{\phi} \) on \( [-\Omega/2, \Omega/2] \), and follow the similar proof seen in [DW].]