Homework 4

1. Let $\{V_j, \phi\}$ be an MRA. So ϕ satisfies the scaling equation

$$\phi(t) = \sqrt{2} \sum_k h(k)\phi(2t-k).$$

Let W_0 be the orthogonal complement of V_0 in V_1, i.e., $V_0 \oplus W_0 = V_1$. Define

$$\psi(t) = \sqrt{2} \sum_k g(k)\phi(2t-k).$$

Show that $\psi \in W_0$ if and only if

$$\overline{H(\gamma)}G(\gamma) + \overline{H(\gamma+1/2)}G(\gamma+1/2) = 0.$$

2. One of the choices of G is that $g(\gamma) = e^{-2\pi i\gamma}\overline{H(\gamma+1/2)}$. Verify that

$$g(k) = (-1)^k \overline{h(1-k)}.$$

3. Verify that another choice of $\{g(k)\}$ can be that

$$g(k) = (-1)^{N-k}\overline{h(N-k)},$$

where is N is an odd integer.