

Homework

1. Assume $\{\phi(t-k)\}_k$ is an ONB of V_0 . Verify that if $f \in V_0$, then $f(t-n) \in V_0$ for all n .
2. If $\{\phi(t-k)\}_n$ is an orthonormal set, show that $\{\phi_{jk}\}_k$ is also an orthonormal set.
3. Let $\{h(k)\}$ be a sequence generating the scaling equation ϕ , i.e.,

$$\phi(t) = \sqrt{2} \sum_k h(k) \phi(2t-k).$$

Verify that

$$(i) \hat{\phi}(\gamma) = \frac{1}{\sqrt{2}} H\left(\frac{\gamma}{2}\right) \hat{\phi}\left(\frac{\gamma}{2}\right)$$

(ii) Deduce that $\sum_k h(k) = \sqrt{2}$ (using (i) and the fact that $\sum_k h(k) = H(0)$, where

$$H(\gamma) \equiv \sum_k h(k) e^{-2\pi i \gamma k}$$