Homework 2

1. Assume that a Gaussian (window) function achieves the bound of the uncertainty principle. Explain why one can not form a Gabor basis with Gaussian windows.

2. Let $\{x_n\}$ be a frame of $\mathcal{H}$, and let S be the corresponding frame operator. Prove that

$$\forall f \in \mathcal{H}, \quad \sum_n \|\langle f, S^{-1}x_n\rangle\|^2 \leq \sum_n |c(n)|^2,$$

where $\{c(n)\}$ are all possible coefficients such that $f = \sum_n c_n x_n$ for a given $f \in \mathcal{H}$. What does this mean?

3. Let $\{x_n\}$ be a frame for $\mathcal{H}$ with the coefficient mapping $U : \mathcal{H} \to l^2$. Suppose that V^* is a bounded left inverse of U. Show that $\{x_n^* \equiv V^* e_n\}$ is a dual frame of $\{x_n\}$.

4. Let S be a frame operator defined by a Gabor frame $\{g_{m,n}\}$. Let T_n and E_m be the translation and modulation operators defined by $T_n f(t) = f(t - nT)$ and $E_m f = f(t)e^{2\pi imt/N}$, respectively. Verify that

$$S(E_m T_n h) = E_m T_n (Sh), \quad \forall h \in \mathcal{H},$$

and deduce that $S^{-1}(E_m T_n g) = E_m T_n (S^{-1} g)$. What does this last equation imply?

5. (Optional) Can you think of an example (about the application of frames) where the use of alternative (non-standard) dual frames is demanded?

6. (Optional) A transversal filter can in fact be formulated in the frame context. Can you provide a frame theoretical formulation of a transversal filter?