Final Review

Math 228

Fall, 2008

1. Problems form Mid-term Review.
2. Find the directional derivative of $f(x, y) = x^2y + \sqrt{y}$ at the point (2,1), in the direction of $\mathbf{v} = \langle 4, 9/2 \rangle$. Is this (the answer you obtain) the maximum rate of change of f? Why?
3. Find the local maximum and minimum values and saddle points of the function $f(x, y) = 3xy - x^2y - xy^2$.
4. Let $\mathbf{F}(x, y) = (2x + y^2 + 3x^2y)\mathbf{i} + (2xy + x^3 + 3y^2)\mathbf{j}$. Show that the line integral $\int_c \mathbf{F} \cdot d\mathbf{r}$ is independent of path, and then find the value of the integral from (0,1) to (5,3).
5. (a) Rewrite the triple integral

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} dzdydx$$

in the order of $dxdydz$.

(b) Establish the integral (do not solve it) using cylindrical coordinates for the volume of the region D bounded by the paraboliods $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.

6. (Green's Theorem) Use Green's Theorem to find the work done by the force filed $\mathbf{F}(x, y) = \langle x^2y, -xy^2 \rangle$ in moving a particle along a circular curve C: $x^2 + y^2 = 4$ with counterclockwise orientation.

[Hint: The work is the line integral of the force field along the curve C]

7. (Surface area) Find the surface area of the portion of the cone $z = 2\sqrt{x^2 + y^2}$ between $z = 2$ and $z = 6$.
8. (Stoke's Theorem) Let $\mathbf{F} = x^2y^3\mathbf{i} + \mathbf{j} + \mathbf{zk}$, and C be the intersection of the cylinder $x^2 + y^2 = 4$ and the hemisphere $x^2 + y^2 + z^2 = 16$, $z \geq 0$, counterclockwise when viewed from above. Use the surface integral in Stoke's Theorem to calculate the circulation of the field F around the curve C.
9. (Divergence Theorem) Let $\mathbf{F} = y\mathbf{i} + \mathbf{xyj} - \mathbf{zk}$, and D be the solid region inside cylinder $x^2 + y^2 \leq 4$ between the planes $z = 0$ and the paraboloid $z = x^2 + y^2$. Use the Divergence Theorem to find the outward flux of $\mathbf{F}$ over the boundary surface S of D, i.e., to calculate the surface integral of $\mathbf{F}$ in the normal direction: $\int\int_S \mathbf{F} \cdot \mathbf{n} d\sigma$.