Math 430 Operations Research Fall 2003 Homework

Homework I

This homework is due Thursday, September 11 at the beginning of the class. If you have any questions send me e-mail (`serkan@math.sfsu.edu`) or call me (338 7723).

1. (Exercise 1.14) A company produces and sells two different products. The demand for each product is unlimited, but the company is constrained by cash availability and machine capacity.

Each unit of the first and second product requires 3 and 4 machine hours, respectively. There are $20,000$ machine hours available in the current production period. The production costs are \$3 and \$2 per unit for the first and second product, respectively. The selling prices of the first and second product are \$6 and \$5.40 per unit, respectively. The available cash is $\$4,000$; furthermore, 45% of the sales revenues from the first product and 30% of the sales revenues from the second product will be made available to finance operations during the current period.

a) Formulate a linear programming problem that aims at maximizing net income subject to the cash availability and machine capacity limitations.

b) Solve the problem graphically to obtain an optimal solution.

c) Suppose that the company could increase its available machine hours by $2,000$, after spending \$400 for certain repairs. Should the investment be made?

2. (Exercise 1.9) Consider a school district with I neighborhoods, J schools, and G grades at each school. Each school j has a capacity of C_{jg} for grade g. In each neighborhood i, the student population of grade g is S_{ig}. Finally, the distance of school j to from neighborhood i is d_{ij}. Formulate a linear programming problem whose objective is to assign all students to schools, while minimizing the total distance traveled by all students. (You may ignore the fact that numbers of students must be integer.)
Notes: I, J, G, C_{jg}, S_{ig}, d_{ij} are the *parameters* of this problem. They are constants that will be provided by an actual problem instance. What this means is the following: the parameters will have different values depending on the school district. If we are solving this problem for East Bay school district the parameters will have certain values; on the other hand, if we are solving the same problem for San Francisco school district the parameter values will change. However, the model itself, and therefore your formulation, will be identical for both districts.

3. Lizzie's Dairy produces cream cheese and cottage cheese. Milk and cream are blended to produce these two products. Both high-fat and low-fat milk could be used to produce these two cheeses. High-fat milk is 60% fat; low-fat milk is 30% fat. The milk used to produce cream cheese must average at least 50% fat and that for cottage cheese at least 35% fat. At least 40% (by weight) of the inputs to the cream cheese and at least 20% (by weight) of the inputs to the cottage cheese must be cream. Both cottage cheese and cream cheese are produced by putting milk and cream though the cheese machine. It costs 40 cents to process 1 lb inputs into a pound of cream cheese. An it costs 40 cents to produce 1 lb of cottage cheese, but every pound of input for cottage cheese yields 0.9 lb of cottage cheese. Cream can be produced by

evaporating high-fat and low-fat milk. It costs 35 cents to evaporate 1 lb of high-fat milk. Each pound of high-fat milk yields 0.6 lb of cream. It also costs 35 cents to evaporate each pound of low-fat milk, and this yields 0.3 lb cream. Each day, up to 3000 of input can be sent through the cheese machine. Each day, at least 1000 lb of cottage cheese and 1000 lb of creamcheese must be produced. Up to 1500 lb of cream cheese and 2000 lb of cottage cheese can be sold each day. Cottage cheese is sold for \$1.20/lb and cream cheese \$1.50/ lb. High-fat milk costs 80 cents per pound and low-fat milk costs 40 cents per pound. The evaporator can process at most 2000 lb of milk per day. Formulate an LP that can be used to maximize Lizzie's daily profit.

4. Convert the following linear program to standard form:

$$\begin{array}{lccccc} min & 17x_1 & +13x_2 & +x_3 & & \\ s.t. & 3x_1 & +5x_2 & & & \geq 11 \\ & x_1 & +7x_2 & +x_3 & & = 40 \\ & |x_1 & +x_2| & & & \leq 10 \\ & & & & x_2, x_3 & \geq 0 \end{array}$$