First Exam, Math 245

1. Solve the initial value problem

\[ x^2 y + xy' = 1, \quad x > 0, \quad y(1) = 2. \]

2. Show that the equation

\[ 2xy^3 + e^x + (3x^2y^2 + \sin y) \frac{dy}{dx} = 0 \]

is exact and find the general solution.

3. A tank contains 1000 L of brine with 15 kg of dissolved salt. Pure water enters at 10 L/minute. The mixture drains from the tank at the same rate. How much salt \( x(t) \) is in the tank after \( t \) minutes?

4. Find \( A^{-1} \) if

\[ A = \]

5. Find the general solution of the homogeneous differential equation

\[ 2xy + x^2 \frac{dy}{dx} = y^2. \]