Math 301-2: Exploration and Proof
In-class assignment
Thursday October 15, 2009

Recursion

Last time we defined $(x_j)_{j=1}^{\infty}$ as a **sequence** of integers. The index j may start at another integer m different than 1 to obtain: $(x_j)_{j=m}^{\infty}$.

Example 1: Let $x_j = j^3 + j$, compute x_j for *j=1,2,3,4,5,100*

Recall: *m* divides *n* is there exists an integer *j* such that *n=jm.* Denote the integer *j* by $\frac{n}{m}$.

Example 2: Let *m* be a natural number. Define a sequence as follows:

(i) Define $x_1 = m$

(ii) Assume that x_n, define

$$x_{n+1} = \begin{cases} \frac{x_n}{2}, if\ 2\ divides\ x_n \\ 3x_n + 1\ otherwise \end{cases}$$

Compute x_n for n=1-10. Can you propose a general formula?

Example 3:
Show that for all natural numbers *k*, $5^{2k} - 1$ is divisible by 24.

Example 4
Show that for all natural numbers *k*, $2^{2k+1} + 1$ is divisible by 3