\[ P[-z_{\alpha/2} \leq \frac{X - p}{\sqrt{p(1-p)/n}} \leq z_{\alpha/2}] \approx 1 - \alpha \]

Consider

\[ \left| \frac{X - p}{\sqrt{p(1-p)/n}} \right| \leq z_{\alpha/2}. \]

This inequality is equivalent to

\[ (\frac{X}{n} - p)^2 - \frac{z_{\alpha/2}^2 p(1-p)}{n} \leq 0. \]

Now

\[
(\frac{X}{n} - p)^2 - \frac{z_{\alpha/2}^2 p(1-p)}{n} \\
= (1 + \frac{z_{\alpha/2}^2}{n})p^2 - (2 \frac{X}{n} + \frac{z_{\alpha/2}^2}{n})p + (\frac{X}{n})^2.
\]

Using the quadratic formula and solving for \( p \) we obtain

\[
\frac{\frac{X}{n} + \frac{z_{\alpha/2}^2}{2n} \pm z_{\alpha/2} \sqrt{\frac{1}{n}(1-\frac{X}{n}) + \frac{z_{\alpha/2}^2}{4n^2}}}{1 + \frac{z_{\alpha/2}^2}{n}}.
\]