Abstract: In this paper we describe the evolution of a task that we used with two cohorts of participants in a professional development program called Revitalizing Algebra (REAL). We first discuss our goals and describe the participants, and then we describe the construction the task followed by teacher responses. We reflect on the different iterations of the tasks and their impact on the teachers’ thinking and practice. We describe how some teachers, influenced by this task, were able to consider changes in their schools and how the school and department cultures contributed to such change.
In this paper we describe the evolution of a task that we used with two cohorts of participants in a professional development program called Revitalizing Algebra (REAL). We first discuss our goals and describe the participants, and then we describe the construction the task followed by teacher responses. We reflect on the different iterations of the tasks and their impact on the teachers’ thinking and practice. We describe how some teachers, influenced by this task, were able to consider changes in their schools and comment on some the conditions for successful change.

The Teachers of Revitalizing Algebra

REAL (Revitalizing Algebra) is a National Science Foundation Mathematics and Science Partnership between San Francisco State University (SFSU) and five local Northern California school districts. The goal of the partnership is to improve the performance of all students (particularly minority students) in algebra, both in K-12 schools and in college. With the participating mathematics departments, we selected teacher leaders on the basis of their interest and their leadership potential.

We did two years of work with two cohorts, each of 27 participants. Each cohort consisted of 9 secondary algebra teachers, 9 graduate student instructors for remedial algebra at SFSU, and 9 mathematics majors aspiring to be teachers. In addition to participating in the REAL class, each undergraduate mathematics major worked ten hours per week in an algebra class of one of the participating secondary teachers. The first year included an intensive two-semester weekly class and a three-week summer session. In this paper we will focus on the secondary teachers.

The second year, secondary teacher leaders worked with teams of teachers in their own departments at their schools using common daily preparation time (paid for by the REAL project) to help each other examine the day-to-day effectiveness of their teaching. They also began to wrestle with longer-term pedagogical issues as a basis for permanent growth in their department teaching culture. We had introduced the leaders to the ideas of lesson study, along with some examples (e.g. Fernandez and Yoshida, 2004; Lewis, 2005), but we let teams find a structure that suited their local conditions.

Basic Principles of Revitalizing Algebra

Our plan for REAL takes into consideration what research and our own experience tell us about the qualities of effective professional development. We were aware of the ineffectiveness of “top-down” educational movements. Influential professional development needs to be highly adaptable, long-term, and relevant to the local conditions of the teachers, yet one cannot encourage growth without some underlying basic principles. We now discuss some basic principles, making frequent references and connections to the Taken As Shared document (Watson and Mason, this
volume) in the interest of brevity.

First, we based our work on considerable research about the complexity of learning. Watson and Mason (this volume) describe theories of learning based on the work on Vygotsky, Piaget and others in which learners are challenged with actively reconciling new ideas and old ideas, intuitive understandings with conventional understandings. In brief, our first principle is:

1. Learners understand formal mathematical ideas by connecting them to their own intuitions and constructed understandings.

This view could be seen as Piagetian, requiring individual construction of meaning, or Vygotskian, requiring dialogue to relate spontaneous and scientific conceptualizations. However, we are more interested in the bridge provided in the work of Skemp (1987), particularly his notion of instrumental versus relational understanding, where the teachers on the course may have a view of mathematics as instrumental because that view was adequate in their previous experience.

Second, Watson and Mason note the importance of milieu, which refers both to mathematical and social environment. In American public school settings, the student body is usually highly diverse in race, gender and culture, so this environment is necessarily influenced by such diversity. Our second principle is

2. Assumptions about race, gender and culture affect the dynamics of a classroom, and must be consciously considered by teachers.

For our chosen content focus, first-year algebra, race, gender and culture are essential parts of the milieu to be wrestled with. The widespread American system of tracking has resulted in a large proportion of non-white, poor students being segregated into mostly non-white, low-track, first-year algebra classes. These classes can offer an impoverished algebraic experience which we needed to address through REAL. We acknowledged the power of racial, gender and cultural discrimination in education and took as part of our charge the important and delicate task of addressing it. We were influenced here by the work of Weissglass (1997) and Delpit (1988) among others. We worked to create opportunities for participants to think about the powerful effects of society’s construction of race, the cultural conflicts that often fall along racial lines, and the influences of these forces in the classroom. Further than this, we wanted teachers to experience ways of understanding algebra which would provide pedagogic tools with which to make a social difference.

In observing and working with participants, we looked for change that was consistent with these two principles, but we were flexible about how teachers changed their practice. As a result, we consciously avoided declaring correct strategies and philosophies for the program. We felt the insights that teachers derived voluntarily from their work would be more meaningful and last longer than ideas pushed on them with the force of our status and authority. Accordingly, we asked teachers to perform tasks (in groups and alone) that challenged their intuitions, presented them with new ideas, and
(we hoped) inspired them to construct new personal and collective understandings of teaching strategies and philosophies. We found that some participants found it hard to accept and adapt to our non-prescriptive approach, while others came to thrive with this approach. While we were flexible in our expectations of what the participants did, the content and pedagogy of the program was based on key themes that evolved from our basic principles about learning and social justice, our shared observations from our weekly classroom visits, and our own teaching experience.

We will focus, here, on the theme of encouraging teachers to identify and use Big Ideas to organize and focus their teaching. The notion of using Big Ideas is certainly not original to us. For example, Papert (2000) and Cuoco, Goldenberg and Mark (1997) exhort people to reorganize school curricula around their proposed big ideas. However we were unable to locate research case studies of work with teachers where the goal is for the teachers to identify and use Big Ideas. By giving teachers this experience in an algebraic context, we believed they might at the very least come to see algebra in new ways, and at best become empowered to take other aspects of the curriculum and reshape them to make more sense for teaching and learning. For the rest of this paper we focus on the ‘Big Ideas’ task and leave discussion of other themes to other articles (e.g. Hsu, Kysh and Resek, 2007).

In this article we focus on the evolution of the Big Ideas task as a case study of a real-life struggle in the field to reconcile a fundamental tension in teacher education. As described by Watson and Mason (this volume), teachers want ideas they can use immediately in their classroom, and this goal makes deeper personal development a challenge. Furthermore, Brousseau (1997) points out that there is an unconscious tendency for leaders to cue participants to perform in ways that seem to indicate deep learning but merely indicate the desire for both sides to feel that successful change has occurred. Indeed, as we shall describe, the course participants appeared to want us to do this. We wanted deep change in their structuring of algebra; they wanted to know more exactly what we wanted them to do. In this specific case, we wanted our teachers to develop a skill and desire to look for the deep structure of the algebra curriculum so that they could exploit the important connections throughout the material and emphasize the central themes of algebra. This seeking of deep structure is itself a skill we hoped they would develop. Teachers, on the other hand, wanted us to tell them what we thought the central themes and connections were. It was a temptation to simply tell them the answers we wanted to hear from them, but instead we struggled to develop and adapt the task so they would develop an ability to do their own inquiry. We imagined this would be a lasting skill which they could apply to other curricular settings.

**Big Ideas**

**Motivation**

On our initial school visits, we noticed that students were concerned only about
getting the answers. There was little or no discussion of reasons or underlying principles. The teacher ‘stamped’ completed homework papers, ‘went over’ the answers, and then introduced students to new procedures, which they practiced. When we discussed these issues with teachers, they talked about not having time to discuss broader ideas. They felt a need to expose students to every procedure that might be on the state test.

Big Ideas was one of several themes through which we worked with teachers to change this classroom pattern. Our goal was to focus teachers on big ideas which would lead to or explain a number of procedures they were teaching. For instance, several of the teachers did not realize that knowing the definition for positive integral exponents is all that is needed to generate the exponent rules; nor did many realize that combining like terms and the common American mnemonic for multiplying binomials ‘FOIL’ (find the products of the First, Outside, Inside and Last terms) are both instances of the distributive property. Thus, students were learning numerous unrelated procedures which could have been derived from a few root ideas. We believed that an approach which focused on root ideas would make mathematics seem more sensible to students, would make it easier for them to perform algebraic manipulations a year or two after instruction, and would make mathematics more interesting because of the increase in meaning and skill.

Evolution of The Big Ideas Tasks

When we first introduced a discussion of Big Ideas, we encountered complications that seemed connected to the lack of precision of our prompts and ineffective facilitation of discussions. But after trying a variety of prompts and tasks with two cohorts, evaluating and redesigning as we shall describe, and after discussion with other professional developers, we suspect that this experience offers insights into how teachers grapple with some important issues.

First, we posed what we naively thought was a straightforward task. Each group of four or five participants was to come up with a short list of Big Ideas for Algebra 1.

A Big Idea is defined as a topic or idea that:

A. connects to different parts of the curriculum,
B. when understood serves as a basis for understanding other topics, and
C. is specific enough to be used to make choices about curriculum.

We intended that the groups would then negotiate with each other over which ideas were more essential than others in order to condense their lists into a single consensus list. With skillful facilitation, this process would require them to discuss which ideas were more powerful and closely connected with the rest of the algebra curriculum. We then intended that they would work in smaller units to use Big Ideas as the foundation for a series of lesson plans.

First Try: With Cohort I we began work on big ideas in the first class of the second semester. Our specific prompt was, “Since there is not time to teach everything, we need to isolate the 5 or 6 main ideas of algebra and focus on topics close to those ideas.
(Notice that we underemphasized aspect (B)). We chose the target numbers 5 or 6 because that seemed restrictive enough to force negotiation over what items to include and in the process make some assumptions visible.

We had few expectations of specific correct answers for such a list. We wanted to develop and practice the process of finding big ideas, rather than create an actual set of Big Ideas. Of course, we certainly had opinions about what would count as a legitimate Big Idea. We thought Big Ideas might include graphing, rules for manipulating algebraic statements, equivalent equations/solving equations, proportional reasoning/percent, linear modeling, moving between different representations of functions, rates of change, or solving systems of equations. On the other hand we would have taken issue with absolute value (too narrow), exponents (too generic), factoring polynomials (a topic emphasized for historical reasons).

In practice we had no success in reaching consensus on a compact list. Teachers seemed reluctant to challenge each other’s choices, and the tendency was to either create a list that was the union of all suggestions, or to summarize all suggestions with very broad topics such as “problem solving” or “representations.” At the close of the session an anonymous comment card said that the three course leaders seemed frustrated that they were not coming up with “the right answers”. We had been frustrated, of course, but that was about the nature of answers rather than an expectation of particular answers.

Revision #1. We speculated internally that by not focusing on aspect (B) we had failed to provide a foundation for deciding which ideas were truly big. At the beginning of the next session we responded that we thought that we had not asked the “right question,” and we followed up by asking small groups to look at a specific chapter on polynomials and find the ‘core’ ideas; what could be left out and what should be emphasized? We also emphasized the importance of choosing ideas that would serve as a basis for understanding other topics (B).

Our restatement appeared to help. After a wandering discussion, at the end of that day the participants collectively decided the two core ideas behind polynomials were the distributive property and the definition of exponentiation as repeated multiplication. They also said that representations and problem solving were two ‘lurking’ big ideas. These ideas were pushed by a couple of teachers in particular, but they seemed to have the agreement of the whole class. We could not tell whether the whole class accepted these findings or whether they merely followed dominant personalities.

We were very pleased with this outcome and felt that the class generally was ready to try using their short list as a basis for developing lessons. We asked groups of three to work on developing a lesson plan around the core ideas with the ‘lurking big ideas’ at the basis of their plan. Then, as homework, we asked each participant to devise an individual outline for the chapter on polynomials. We asked for enough detail so a colleague could read it and teach it. We got very mixed results. Many presented one activity without an outline of the overarching Big Idea. Only a fifth of them produced an outline we found acceptable. During the next session we asked two participants who had
reasonable plans to share them and then we put the groups back to work to design lesson plans that build on the core ideas and have the ‘lurking ideas’ integrated throughout. The anonymous comments for the day showed that a number of participants still did not appear to understand that some ideas (such as the distributive law) generate other ideas (such as combining like terms). They also wondered where we were going with this subject and why we did not let them formulate complete lesson plans. The next day we had a short class discussion on the place of teaching shortcuts versus use of core ideas in an algebra classroom, and we explained that we wanted to change their thinking about teaching, not merely send them off with some lesson plans. As before, the task did not go as planned, and it was clear that most of the teachers still did not grasp the point of Big Ideas.

Revision #2. We were not sure what the obstacle was, and we were reluctant to abandon our approach because of our commitment to the underlying principles. We decided to give more active feedback about which proposed ideas seemed Bigger than others. About five weeks after Revision #1, in a homework assignment, we handed out a traditional algebra text and asked groups to identify the big ideas in the specific chapter about two equations with two unknowns. In the following class, participants worked in groups to identify with the core ideas. Once again, they seemed to include the union of the group members’ ideas rather than working for an essential intersection. The quality of ideas was so spread out that it seemed impossible for session leaders to respond in a way that valued the contributions, particularly since we had tried this task several times already. Comment cards that day criticized the facilitators for not pushing the groups into consensus. There was a lingering sense in the class that, once again, we had a correct list in mind but that they were not given enough information to produce it for us. Again we responded that we were not looking for the ‘true list,’ we wanted a good discussion to develop their thinking about priorities in their algebra curriculum.

Revision #3. At this point we had almost given up on the task. We began to think there was some fundamental problem with the task, some flaw in our design and facilitation, or perhaps that the topic of big ideas in algebra was just too hard for our current group. It was out of a sense of obligation to school students that we decided to revisit Big Ideas during the summer session. We planned to be as specific as possible about all three aspects of a Big Idea: (A), (B) and (C) and we decided to be brutally honest in our response.

On Day 2 of our summer session, we asked groups of 5 or 6 to suggest core ideas for an algebra course. We anticipated resistance, suspicion, frustration and confusion. To our astonishment, in a relatively short time, the groups constructed good lists of ideas, and they reached consensus through a reasonable and skilled discussion. Instead of frustration and accusations of obscurity, the overwhelming mood was casual. The comment cards from that day said things such as “Enough is enough!” and “Why must we keep doing this?” Some time between the spring semester and the summer session, the notion of Big Ideas in algebra had gone from being incomprehensible to obvious.

In the end-of-summer evaluation “Big Ideas” got a low rating. Participants said
that one problem with the discussion was that they were all using different textbooks, and they needed to understand each others’ programs better 1. A few felt we were pushing them to eliminate most of the textbook material and they did not agree with this aim. But others felt that perhaps they needed to go through all the muddled discussions to get to their current understandings. One cannot take student evaluations of tasks as the final measure of task quality. Our aim that the curriculum should be seen as orientated around big ideas seemed to have been achieved. However, we did use the feedback to modify the task to respond to the most common student suggestions.

Revision #4 (Cohort II) In response to the comments from Cohort I, we made several changes for Cohort II. First, we spent time at the beginning of the year asking participants to familiarize each other with their algebra books and programs. Second, we tried to define more clearly what we wanted on our first assignment. When introducing it, we spoke of ‘root ideas’ which generate other ideas, and we emphasized all three aspects: (A) (B) and (C). Third, we explicitly said we were interested in content ideas. We thought that problem-solving and applications are processes which are overarching and would distract from their task; we told them they need not mention such processes explicitly. Apart from this, we used a similar set of activities and assignments, but with the modifications of Revision #3.

The results were similar. Participants said they did not understand what we wanted. Groups offered chapter headings rather than big mathematical ideas. They did not enjoy the discussions, and accused us of having a hidden agenda. By the time summer came around, we had almost given up. But amazingly, just as had happened the year before, they did well in identifying big ideas and said similar things to Cohort I: “Enough is enough!” and “Why do you keep making us do this?”

Discussions with colleagues at U.C. Berkeley and Michigan State University indicated that others have also noticed anecdotally a leap of understanding at the end of the first year, in that identifying the essential themes of a course goes from being a frustrating and confusing task to a routine and obvious one. We cannot fully explain this phenomenon, but we speculate that it fits a pattern for learning that can be observed in how school students respond to some areas of mathematics. We further speculate that simplifying the task to avoid frustration might not have led to this final realization.

Theoretical and Practical Implications for Teacher Education

We have described a task sequence and its direct outcomes in our classrooms. However, we also believed that to make lasting change at our partner schools, teachers would need to voluntarily change the working culture of their departments. For example, we wanted to establish the importance of Big Ideas not only in the minds of individual teachers, but in the discourse and practice of the departments as a whole. In this final

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1 The teachers with whom we worked were highly bound to textbook use, and often saw the curriculum as the contents of specific texts.
section, we will describe the influence these ideas had on different secondary mathematics departments, based on the data we have collected throughout the project. We shall close with some more general observations on teacher change in our project.

During their second year in the project, REAL paid for released time for the secondary teachers (whom we call “lead teachers”) so they could work with most of the rest of their department colleagues in a common daily preparation period (with one additional preparation period for the lead teachers). We had hoped this would give teachers the time to work seriously on their craft and to consider the principles and ideas we had worked on in REAL. We visited these common preparation periods regularly.

We saw the influence of the idea of Big Ideas in the department daily meetings at every school. Lead teachers may not have used our tasks word-for-word (indeed we intentionally did not give them written prompts or templates for the Big Ideas tasks), but we saw teachers seeking out big ideas using them to reorganize their curricula. All of our partner schools put a great deal of their initial effort into rethinking and restructuring the mathematics curriculum. There seemed to be a feeling of new-found freedom with the realization that they could change the content and emphasis of what they taught. These discussions were framed in terms of “Big Ideas,” even though departments usually did not come to a consensus on an explicit and complete list of Big Ideas. For instance, one school decided that graphing was a cornerstone of algebra, and reworked their curriculum so that it came at the beginning of the course and was referenced throughout. Another school pruned some topics from their syllabus that they felt were not central to the course, even though the topics were on the State test, in order to focus more effort on what they considered vital. On the whole, our partner schools all made changes to curriculum, from adding supplementary activities, to fully rearranging the algebra course.

Furthermore, there were some encouraging anecdotes that showed teachers were reworking activities based on Big Ideas beyond the direct department work. For instance, a lead teacher at a third school said she was inspired by a conversation in the daily preparation period about the centrality of the relation of graphs to their equations. She devised an introduction to solving two equations with two unknowns. She first asked students to find as many pairs of numbers which added to 15 as they could. Then she said in addition to adding to 15, what if the difference had to be 3? Still another lead teacher, thinking about students’ difficulties in solving absolute value inequalities, moved from a strictly set-theoretic approach to a graphical approach with a number line. She also included real-life examples of disjunctions and conjunctions using sets of students (“sophomores” and “dog lovers”).

The teachers in REAL had resisted when pushed to think deeply and share with their peers their thinking about what mathematics was important to teach and why, and how they might best teach it. Some teachers had come to REAL thinking that the project directors had already worked all of that out. Similarly, students generally do not question the norm that giving the right answer is what it takes to be successful in school, and resist when asked to think more deeply themselves. In fact, what REAL advocated for both teachers and students was the value of the process of thinking and struggling with questions. We could be more explicit in our future work about getting teachers to reflect on this parallel experience in their learning to shape the learning experiences they create
for their students.

We had to learn patience. In retrospect, it seems difficult and unrealistic to ask teachers to change their practice dramatically in the middle of a school year. The fall and spring semesters were marked by slow, cautious change and an opening of minds, followed by great leaps of attitude and ambition during the summer session after the school year, followed by slower but more visible change during the following school year. We could find no easy way around this dynamic.

Second, we intentionally constructed the program’s structure and incentives to allow teachers to change at their own pace. Still, we needed patience in the second year waiting for our lead teachers’ visible changes of heart, mind, and speech to result in changes in the classroom. A small number of teachers were not influenced by the program, but the majority of participants showed their changes of attitude in their visible work, in their rhetoric, in their discussions with their peers, and in their private interviews with the outside evaluator. Indeed, most teachers did try different things in the classroom, and, as discussed above, some had been profoundly influenced. But many of them would try to teach differently in one class, then return to their previous practice for a few classes, and then try something different again (often when one of us came to visit).

We have focused our attention on the lead teachers in this paper, most of whom were influenced to change, though sometimes not as quickly as we hoped for. In contrast, we are disappointed in the progress of their school colleagues. Simply put, despite their daily meetings with lead teachers, we saw very little change in the practice or attitude of the colleagues. The exceptions were where the whole department agreed to change the curriculum. It will take another, different article to analyze this particular phenomenon, but it is striking to us that out of six high schools and two middle schools, the two schools that did show some effect on colleagues were the two that most closely imitated the sequencing and activities of the REAL workshop in their own daily meetings. In most cases the activities of our project pushed and challenged our lead teachers in ways not achieved in their own second-year meetings, which were sometimes dominated by simple curriculum reordering and sharing tips for teaching particular topics. Remembering the lesson about patience we learned from the lead teachers, we still hope we will see changes in their colleagues’ practices in the coming years.

Our key original goal was to improve the performance of all students in algebra, particularly minority students. Our work on Big Ideas was designed to help teachers focus their classes on a smaller number of deeper topics in algebra, so that students can see procedures developed from a few key ideas. That is, procedures will make sense to students and algebra will be intellectually interesting. While this focus can benefit all students, for students who belong to the dominant culture and who buy into the system of school, the use of Big Ideas enhances their learning but is not usually necessary to their success. For students who are not participating, play by different rules, don’t have confidence in their math abilities, don’t buy into the value of the system, don’t have extrinsic reasons for putting an effort into learning what appears to be meaningless from someone they don’t trust, in short are not succeeding, a Big Ideas approach is necessary.
They need a reason to get intellectually involved, and an approach that is challenging and accessible is one way to do that. Furthermore, many of our target students are repeating algebra, and have already been taught in a non-Big Ideas way; a fresh approach may help some students.

We have been collecting data on algebra passing rates in our project schools, and we can report the changes over the course of our project work (either four or five years, depending on cohort). In six school sites where we have continued to work past the original two-year project, five increased African-American passing rates in first-year algebra (+0.0%, +8.8%, +13.7%, +15.0%, +17.6%, and +27.7%) and five increased their overall passing rate (respectively: +16.0%, +6.3%, -4.0%, +11.5%, +5.8%, and +14.3%). In the three schools where we have not continued to work (and where our evaluator observed us to have little impact on teaching practice), there was little positive progress in African-American passing rates (-14.7%, -1.5%, +1.4%) or overall passing rates (respectively: -3.3%, -8.1%, -10.6%). It is difficult to establish how much Big Ideas and our other work contributed to the movement in failure rates, but we find the data encouraging.

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