4. Use Fourier-Motzkin elimination on P to eliminate $x_1$. P is the polygon described by

$$
\begin{bmatrix}
-1 & -4 \\
-2 & -1 \\
1 & -2 \\
1 & 0 \\
2 & 1 \\
-2 & 6 \\
-6 & -1
\end{bmatrix} \leq \begin{bmatrix}
-9 \\
-4 \\
0 \\
4 \\
11 \\
17 \\
-6
\end{bmatrix}.
$$

We multiply the matrix out and obtain the following inequalities:

\[
\begin{align*}
-x_1 - 4x_2 &\leq -9 \Rightarrow x_1 \geq 9 - 4x_2 \\
-2x_1 - x_2 &\leq -4 \Rightarrow x_1 \geq \frac{2}{3} - 2 \\
x_1 - 2x_2 &\leq 0 \Rightarrow x_1 \leq 2x_2 \\
x_1 &\leq 4 \\
2x_1 + x_2 &\leq 11 \Rightarrow x_1 \leq \frac{11}{2} - \frac{x_2}{2} \\
-2x_1 + 6x_2 &\leq 17 \Rightarrow x_1 \geq 3x_2 - 17 \\
-6x_1 - x_2 &\leq -6 \Rightarrow x_1 \geq 1 - \frac{x_2}{6}
\end{align*}
\]

The first thing to notice is that $x_1 \leq 4$, so we already have an upper bound for $x_1$. We now proceed to completely eliminate $x_1$ using the above inequalities. That is, we’re looking for a lower bound and upper bound for $x_1$ in terms of only $x_2$.

Now we have

$$
\begin{align*}
3x_2 - 17 &\leq x_1 \leq 2x_2 \\
1 - \frac{2x}{3} &\leq x_1 \leq 4 \\
x_1 &\leq \frac{11}{2} - \frac{x_2}{2}
\end{align*}
$$

Next, we characterize $\text{proj}_1(P)$. $\text{proj}_1(P)$ is given by the projection of the polytope onto the $x_2$ axis (then $x_1$ is 0). Now we solve all possible inequalities:

- $9 - 4x_2 \leq 2x_2 \Rightarrow x_2 \geq \frac{3}{2}$
- $9 - 4x_2 \leq 4 \Rightarrow x_2 \geq \frac{5}{4}$
- $9 - 4x_2 \leq \frac{11}{2} - \frac{x_2}{2} \Rightarrow x_2 \geq 1$
- $3x_2 - 17 \leq 2x_2 \Rightarrow x_2 \leq 17$
- $3x_2 - 17 \leq 4 \Rightarrow x_2 \leq 7$
- $3x_2 - 17 \leq \frac{11}{2} - \frac{x_2}{2} \Rightarrow x_2 \leq 7$
- $1 - \frac{5x}{6} \leq 2x_2 \Rightarrow x_2 \geq \frac{6}{13}$
- $1 - \frac{5x}{6} \leq 4 \Rightarrow x_2 \geq -18$
- $1 - \frac{5x}{6} \leq \frac{11}{2} - \frac{x_2}{2} \Rightarrow x_2 \geq \frac{3}{7}$
- $\frac{x}{2} - 2 \leq 2x_2 \Rightarrow x_2 \leq 4$
- $\frac{x}{2} - 2 \leq 4 \Rightarrow x_2 \leq 12$
- $\frac{x}{2} - 2 \leq \frac{11}{2} - \frac{x_2}{2} \Rightarrow x_2 \leq \frac{11}{2}$

Thus, we have $\text{proj}_1(P) = \{ x_2 | \frac{3}{2} \leq x_2 \leq 4 \}$