Can shorten: *dual letters* → leave \( abab \)
* \( abab = ba \)
* \( babab = ab \)

So elements: \( \{ e, a, b, ab, ba, aba \} \)

Why the same?
* \( a^2 = e \)
* \( b^2 = e \)
* \( aba = bab \)

Operation: "stretching wire"

### 3. Geometry

**Kaleidoscope**

### 3. Geometry

**Six reflections**

\[
\begin{align*}
\text{generator: } S & \\
\text{relations: } (ss')^m &= e \\
(aba)^2 &= e \\
(aba)^2 &= e
\end{align*}
\]

**Remark**

* No relation when \( m(s, s') = 0 \)
* \( (ss')^m = e \iff ss's's's's \ldots = s's's's \ldots \)
* No edge \( s \rightarrow s' \), means that \( s, s' \) commute
\((W, S)\) is called a Coxeter system

\(|S|\) is the rank of \((W, S)\).

Think:
- Elts of \(W\): words in the alphabet \(S\), regarding
\[
\frac{uv (\ldots) v}{2m(s, s')} = \frac{uv}{2m(s, s')}
\]
- Operation: gluing words.
  \(\rightarrow\) identity?
  \(\rightarrow\) inverse?

Formally, \(W \cong \mathbb{F}/N\) when
- \(\mathbb{F}\) = free group generated by \(S\)
- \(N\) = normal subgroup generated by \(\{(ss')^m | s, s' \in S\}\)

Examples

\[s \rightarrow W = \langle s | s^2 = 1 \rangle = \{e, s\}\]

\[s_1, s_2 \rightarrow W = \langle s_1, s_2 | s_1^2 = s_2^2 = (s_1 s_2)^3 = 1 \rangle = S_3\]

Exercise: \(W \cong D_{\infty}\) dihedral group

- \(W = \langle s_1, s_2 | s_1^2 = s_2^2 = (s_1 s_2)^3 = 1 \rangle\)

Exercise: \(W \cong S_3\) (Mooij)

\[s_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad x_1 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\]

\[x_1^2 = x_1 x_1 = x_1 x_1\]

\(\triangle\) A group \(W\) can have different presentations as a Coxeter system. (Exercise: \(D_6\))

Important examples

- groups generated by geometric reflections
- groups of symmetries of the regular polytopes
- Weyl groups of root systems/Lie algebras/Lie groups

(Will say more)  (Will say more)

(Will say more)  (Will say more)  (Will say more)