• #5 Give an example of an infinite Coxeter group whose weak order has no infinite anti-chains, and an example of an infinite Coxeter group whose weak order has an infinite anti-chain.

Proof. The Coxeter group generated by $S = \{a, b\}$ with $m(a, b) = \infty$ has no anti-chains of more than two elements since any $w \in (W, S)$ is either in the chain $e \leq_R a \leq_R ab \leq_R aba...$ or $e \leq_R b \leq_R ba \leq_R bab...$ and given three elements two will at least two will be in the same chain.

The Coxeter group generated by $S = \{a, b, c\}$ with $m(a, b) = \infty$, $m(a, c) = \infty$, and $m(b, c) = \infty$ has infinite anti-chains. For example, $\{b, ab, acab, acacb, acacab, \ldots\}$. □