9. Assume by contradiction that there exists an infinite antichain in the Bruhat order of $W$. We will say that a sequence $w_1, w_2, w_3, \ldots$ of elements in $W$ is “good” if $w_j \not\leq w_k$ for $j < k$. In particular every antichain is a “good” sequence. Let’s construct a “good” sequence as follows: Let $w_1$ be an element of least length among all first elements of all “good” sequences. Let $w_2$ be an element of least length among all second elements of all “good” sequences starting with $w_1$. Let $w_3$ be an element of least length among all third elements of all “good” sequences starting with $w_1, w_2$. Continue this process up to infinity, so we get a “good” sequence $w_1, w_2, w_3, \ldots$. Fix reduced expressions for all the elements in this “good” sequence. Since there are only finitely many elements in $S$, there must be infinitely many of these expressions that start with the same letter $s \in S$, say $w_{i_1} = sw'_{i_1}, w_{i_2} = sw'_{i_2}, w_{i_3} = sw'_{i_3}, \ldots$ with $i_1 < i_2 < i_3 < \ldots$. Then by the subword property we have that $w'_{i_j} \not\leq w'_{i_k}$ for $j < k$. Then, again by the subword property, we have that $w_1, w_2, w_3, \ldots, w_{i_1-1}, w'_{i_1}, w'_{i_2}, w'_{i_3}, \ldots$ is a “good” sequence. But $l(w'_{i_1}) = l(w_{i_1}) - 1$, contradicting the choice of $w_{i_1}$. 