Knowing the Bruhat order for $(W, S)$ gives us in particular its graph of covering relations, and each of these covering relations corresponds to a single deletion and thus is an edge in the Bruhat graph. So our task is to reconstruct the edges $(w, w')$ of the Bruhat graph with $l(w') - l(w) > 1$. Any Bruhat edge joins two comparable elements, so it’s enough to look at each interval $[w, w']$ in the Bruhat order and determine whether the edge $(w, w')$ should exist. By induction we can assume we’ve already done so for all proper subintervals of $[w, w']$. 
We claim that, for any Bruhat interval \([w, w']\) with \(l(w') - l(w) = d\), there are exactly \(d\) Bruhat edges from elements of this interval to \(w'\). This will allow us to perform the reconstruction of the Bruhat graph: for each interval \([w, w']\), after having finished with its subintervals, \(w'\) will have either \(d - 1\) or \(d\) edges to it from within the interval; the edge \((w, w')\) should be inserted if and only if there are \(d - 1\).

So, toward proving the claim, first take some reduced word \(s_1 \ldots s_{l(w')}\) for \(w'\), so that it has a subword omitting only \(d\) letters \(s_{i_1}, \ldots, s_{i_d}\) which is a reduced word for \(w\). Omitting any single one of these letters \(s_{i_k}\) yields a word \(v\) with \(w \leq v \leq w'\) such that an edge \((v, w')\) exists in the Bruhat graph. This provides \(d\) edges overall; it remains to show there are no more.

Suppose this didn’t hold; let \([w, w']\) be a counterexample with \(l(w')\) minimal. There is a minimal word \(s_1 \ldots s_{l(w')} = w'\), such that there are strictly more than \(d\) indices \(i\) such that deleting \(s_i\) from this word leaves a word \(v \geq w\) in the Bruhat order. Let \(I\) be the set of these indices.

Consider now the element \(w's_i\), where \(s := s_{l(w')}\); this element satisfies \(w's < w\). Given any \(i \in I \setminus \{l(w')\}\), write \(w'_i\) for the word obtained by deleting \(i\) from \(w'\). This is a reduced word, and it ends in \(s\), so \(w'_i s < w'_i\).

Now, we have two cases, according to whether \(l(w') \in I\), equivalently whether \(ws < w\) or \(ws > w\). If \(ws > w\), then the subword of \(s_1 \ldots s_{l(w')}\) giving \(w\) omits \(s_{l(w')}\), so that for any \(i \in I \setminus \{l(w')\}\), \(w'_i s\) is still a superword of \(w\). There are more than \(d - 1\) elements of \(I \setminus \{l(w')\}\), and thus the interval \([w's, w]\) of length \(d - 1\) constitutes a smaller counterexample. If instead \(ws < w\), then for every \(i \in I\), we have \(w'_i s < ws\) by lifting, and there are more than \(d\) such indices \(i\), so the interval \([w's, ws]\) of length \(d\) is a smaller counterexample. In either case we have a contradiction.