If $w_0$ is an element of a Coxeter group such that $sw_0 < w_0$ for every simple reflection $s \in S$, prove that $w \leq w_0$, for all $w \in W$.

**Proof:** By induction.

Show $w$ is a subword of $w_0$.

Let $e = w_0$.

Then $e \leq w_0$.

Assume true for any $u \in W$ with $l(u) < n$ that $u \leq w_0$.

Let $v \in W$ such that $l(v) = n$.

Then $v = s_1 \cdots s_n$, but $s_i v = s_1 \cdots s_n$.

So $l(s_i v) = n - 1 < n$.

Therefore $s_i v \leq w_0$ by the induction hypothesis.

So we have $s, v \leq w_0$, and $s, w_0 \leq w_0$.

Then by the lifting property we get $v$ or $v \leq w_0$.

Therefore $w \leq w_0$ for all $w \in W$.

Worked with David Bangor.