

San Francisco State University
Syllabus for the written analysis comprehensive MA exam
Preliminary outline

Students will be expected to give precise statements of definitions and standard theorems as well as prove simple propositions from definitions or using standard results. They will also be expected to know the proofs of the principle theorems as well as a body of examples to illustrate the theorems or show why the hypotheses are needed. During the course of the exam a student may be given concrete examples to be analyzed using the definitions and theorems.

Real analysis topics

- The real number system
 - Field axioms
 - Order axioms
 - Suprema and infima
 - Axiom of completeness
 - Archimedean property
- Limits of sequences
 - Basic limit laws
 - Cauchy sequences
 - Completeness of the real number system
 - Subsequences
 - Limits inferior and superior
 - Accumulation points
 - Bolzano-Weierstrass theorem
- Limits of functions
 - Basic limit laws
 - Sequential characterization
- Continuity
 - At a point
 - On an interval
 - Uniform continuity
- Basic theorems for continuous functions
 - Extreme Value Theorem
 - Intermediate Value Theorem
 - A continuous function on a closed bounded interval is uniformly continuous
- Derivatives
 - Definition and interpretations
 - Chain rule
 - Linear approximation of differentiable functions
 - Mean Value Theorem
 - Concavity and the second derivative
 - First and second derivative tests for local extrema

- Taylor approximation and the remainder formulas
- Riemann integral
 - Development of the Riemann integral via upper and lower sums, upper and lower integrals
 - Integrability criteria
 - Existence of the Riemann integral for continuous functions and monotonic functions
 - Fundamental Theorem of Calculus
- Convergence of sequences and series of functions
 - Pointwise convergence
 - Absolute convergence
 - Uniform convergence
 - Convergence of power series
 - Convergence in mean
 - Properties stable under uniform limits
 - Interchanging the order of the operations of limits or summation with integration or differentiation
- Metric spaces
 - Limits of sequences
 - Open/closed sets
 - Compact sets
 - Complete metric spaces
 - Continuity of functions
 - Topological definition of continuity
 - $C[a,b]$ as a metric space

References

1. Abbott, Stephen, *Understanding Analysis*, Springer, Chapters 1-7.
2. Lay, Steven R., *Analysis with an Introduction to Proof*, 4th Edition, Pearson Prentice Hall, Chapters 3-9.
3. Wade, William R., *An Introduction to Analysis*, 3rd Edition, Chapters 1-7, 10.

Complex variables topics

- Complex number system
 - Complex plane
 - Complex arithmetic
 - DeMoivre's theorem
 - Nth roots
 - Geometric interpretation of addition and multiplication
 - Exponential, trigonometric and logarithmic functions
- Complex derivative
 - Cauchy-Riemann conditions

- Chain rule
- Analytic functions
- Harmonic functions
- Harmonic conjugates
- Maximum modulus principle
- Complex integration
 - Line integrals
 - Cauchy's Theorem
 - Cauchy's integral formula
 - Cauchy's estimates
 - Liouville's Theorem
 - Fundamental Theorem of Algebra
- Series expansions
 - Taylor series
 - Laurent series
- Residue theory
 - The Residue Theorem
 - Argument principle
 - Evaluation of improper integrals

References

1. Brown and Churchill, *Complex Variables with Applications*, McGraw Hill, 6th Edition, Chapters 1-7.
2. Marsden and Hoffman, *Basic Complex Analysis*, 3rd Edition, W.H. Freeman, Chapters 1-4.
3. Saff and Snider, *Fundamentals of Complex Analysis*, 3rd Edition, Prentice Hall, Chapters 1-6.