Bijections

Be sure to identify the domain and codomain in each question below.

1. Give an example of a bijection.

2. Give an example of an injection that is not a surjection.

3. Give an example of a surjection that is not an injection.

4. Give an example of a function that is neither injective nor surjective.

5. Can you modify the domain and/or codomain in 2, 3, and 4 to make these functions bijections? How?
Counting and Cardinality

1. Let $A = \{a, b, c, d, e\}$ and let $B = \{1, 2, 3, 4, 5\}$. Create a surjection from $A$ onto $B$. Now, come up with a different function from $A$ into $B$ that is an injection. Are your functions bijections? If yes, can you make a surjection or an injection from $A$ to $B$ that is not a bijection? Why or why not?

2. Let $C = \{10, 11, 12, 13, 14, 15\}$ and let $D = \{10, 12, 14\}$. Can you create both a surjection and an injection from $C$ to $D$? What about from $D$ to $C$? What can you conclude?
3. Let $E$ be a finite set. Let $\mathcal{P}(E)$ be the power set of $E$. What can you say about the cardinalities of $E$ and $\mathcal{P}(E)$?

4. Consider the sets $\mathbb{N}$ and $2\mathbb{N}$ (i.e. the even natural numbers). Which set is bigger, in terms of cardinality? Explain your reasoning.

5. Now, consider $\mathcal{P}(\mathbb{N})$. What can you say about the cardinality of this set in relation to $\mathbb{N}$?
Countable?

Which of the following sets are countable? Explain your reasoning.

1. $\mathbb{Z}$

2. $\mathbb{N} \times \mathbb{N}$

3. $A \cup B$, where $A$ and $B$ are both countable.

4. $A \times B$ where $A$ and $B$ are both countable.

5. $\mathbb{Q}$
Polynomials and Algebraic Numbers

Consider the set of linear polynomials $y = mx + b$ with rational coefficients.

1. How many roots can a linear polynomial have?

2. How many linear polynomials are there?

3. What is the cardinality of the set of roots of linear polynomials?

4. Repeat the questions above for quadratic polynomials $y = ax^2 + bx + c$.

5. Repeat the questions above for polynomials of degree $n$.

6. What is the cardinality of the set of all polynomials with rational coefficients?

7. What is the cardinality of the algebraic numbers?