Algebra - Fun with Calendars.

Algebra is used daily. People encounter algebra every day without realizing it. When someone says he is 5 years older than you, you’re setting up an algebraic equation to solve for an unknown:

\[ x = 5 + \text{your age} \]

Another example, you don’t know the original price of a product, but you know the discount amount, then you can figure out that original amount. There’s a shirt for sale. It is 50% off the original price and costs $10. You can figure out what the original was with a little algebra:

\[ .5 \times \text{original price} = 10 \]
\[ \text{original price} = \frac{10}{.5} = 20 \]

Whenever you’re dealing with numbers and relations between quantities, you’re definitely dealing with math, and most likely with algebra. The groceries are another place where people encounter algebra. Suppose you’re walking down the cereal aisle and you see your favorite in two different sizes:

15 ounces for $3.49
21 ounces for $4.99

In order to find out which box is the better deal, meaning cheaper on a per-ounce basis, you have to use algebra.

For high technology, laptop computer is just an implementation in electrical circuits of a special form of algebra, called Boolean algebra, invented in the 19th century. Ordinary algebra is used to design and manufacture computers, and algebra is
at the heart of computer programming [2]. iPod is a math device at your fingertips. The iPod stores music using sophisticated mathematics built on algebra. The iPod shuffle mechanism uses regular school algebra to order your songs randomly [2].

The activity “Fun with Calendars” uses a fun mathematical puzzle to demonstrate solving a simple linear equation. It is an average math trick, but it's an excellent activity to demonstrate algebraic applications. The activity is appropriate for 6th - 9th graders since the middle school students are familiar with variables, simple linear equations and factoring. I am going to do this lesson with the Presidio kids to help them with variables and setting up equations. The activity shows students the fun side of algebra and that algebra is in our everyday lives. I hope this activity will motivate them to learn math.

**Learning Objectives:**

Assigning variables
Solving simple linear equations
Factoring
Connecting algebra to real-world experience

**Materials Required:**

A monthly calendar
Calculator
Papers and Pencils

**Activity Procedures:**

1. Show the students a calendar and play the trick below with them a couple of times.
Tell the students to choose 4 days that form a square like the four below:

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The students should tell you only the sum of the four days, and you can tell him what the four days are.

The students will say, “How did you do that?”
Answer: “I used algebra.”

2. Demonstrate assigning variables to the different days by writing them on the board. Then you can either let them think about it or you can show them how to solve the equation.

   The teacher: “How does this work? People usually don’t realize when they are using algebra in their daily lives. We just used algebra here. Let’s go over it together:
Let’s pretend that the 4 numbers that you chose were 20, 21, 27, and 28. You add up the four numbers and tell me that the sum is 96.

I make a couple of calculations and tell you the 4 numbers. What kind of calculations do you think I used? “

Let the students guess the calculations, and write those guesses on the board. If they are off, say: “Lets figure that out with algebra.”

“Let’s call the first number n. Then you know that the next number would be n + 1 because it is the next day. What about the next number? It would be n + 7. Why? and the last number would be n + 8. Let’s add our four numbers in terms of n:

\[ n + n + 1 + n + 7 + n + 8 \]

And since our sum is 96 when we added them, we have:

\[ n + n + 1 + n + 7 + n + 8 = 96 \]

Simplifying our equation by adding like terms, we get:

\[ 4n + 16 = 96 \]

How would you solve this equation? Would we subtract 16 from both sides?

\[ 4n = 80 \]

Then we would divide both sides by 4

\[ n = 20 \]

That’s exactly how we solve the puzzle. When you tell me the sum, I subtract 16 then divide by 4. This gives me the first number n, which corresponds to the first day. How do we get the other 3 days?” Let them solve for this.

The teacher asks if anyone knows any easier way or method we can solve this puzzle. If there is someone who knows about it, give him or her opportunity to share with the rest of the class. After that, the teacher will explain an easier method to the whole class.
Easier method:

“Subtracting 16 mentally isn’t so easy. Go back to that equation:

\[ 4n + 16 = 96 \]

I think I see a better way. Factor 4 from the left side of the equation:

\[ 4(n + 4) = 96 \]

Now, I could divide both side by 4:

\[ (n + 4) = 24 \]

Subtract 4 from both sides:

\[ n = 20 \]

That is a lot easier to do mentally. **Divide by four and then subtract 4.**”

Summary: “So how does the puzzle work again? Your friend adds any 4 numbers that form a square on the calendar and tells you the sum. You **divide by four and then subtract 4.** That gives you the first number. You add 1, 7, 8 to get the other numbers. Algebra makes it all possible.”

3. Divide the students into groups of 2 and let them practice solving the puzzle with each other.

   Following 5 minutes or so, ask 3-4 groups what is their sum of 4 numbers and what are the numbers. Let them explain how they get the numbers.

4. Demonstrate creating a different puzzle.

   **Designing your own puzzle**

   Think of creating a different pattern, say 3 in a row diagonally. Have your friend add up any three numbers diagonally left-to-right. For example, the picture below:
But this time we wouldn’t divide by 4 and subtract 4 to find the numbers. So we have to figure out how to solve this.

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Ask the students: “Which number do you think would be the easiest to call n?”

Some students may say: “the first one”, some may say: “the middle one” and other students may say: “the last one”

Teacher: “Let’s check each case together. If we call the first number n, what are the second and the third numbers in terms of n?”

Students: “n + 8 is the middle one and n + 16 is the third one”

Teacher: “If we call the middle number n, what are the first and the last numbers in terms of n?”

Students: “n - 8 and n + 8”

Teacher: “If we call the last number n, what are the first and the middle numbers in terms of n?”

Students: “n - 8 and n - 16”

Teacher: “So comparing these different ways in solving this, which do you think would be the easiest day to call n and why?”

Let the students explain their ideas. After that, the teacher explains: “I think making the day in the middle n would be the simplest approach. Then the first day would be n - 8 and the last day would be n + 8. When we add the three numbers:

\[ n - 8 + n + n + 8 \]

We can see -8 and 8 will cancel out, so it makes our equation simpler

For example: If their sum equals 57, the equation is:
\[ n - 8 + n + n + 8 = 57 \]

-8 and 8 cancel out and we combine like terms

\[ 3n = 57 \]

\[ n = 19 \]

Wow! This one’s really easy. You just divide by 3.

“So how do you solve this puzzle? Your friend adds the 3 numbers and tells you the sum.”

Let 2-3 volunteers answer the question. After that, the teacher summarizes: “you divide by three and that gives you the middle number. You subtract 8 to get one number and add 8 to get the other.”

5. Explain how the students will make their own puzzle.

The teacher: “Now you try one. Design a puzzle of your own.”

Write the steps below as instructions on the board:

a. Describe an interesting pattern on the calendar.

b. Call one of the days \( n \).

c. Write the other days in terms of \( n \).

d. Add up the days (in terms of \( n \)) and write it equal to its sum. (The actual number)

e. Figure out the puzzle’s solution by actually solving the equation.

6. Give each group of 2 students a monthly calendar and let them try their own puzzle for 10 minutes.

7. Combine each two groups together, and let them share their own puzzles with the group.

The teacher goes around to help and evaluate student-designed puzzles. Observe as they play their puzzles.
Extension:

1. The trick: Take any calendar. Choose 9 days that form a square like the one below.

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Add all nine numbers and then divide by the number in the middle. You always get nine. Explain how this works and why this works.

2. Where else might this work? Could you modify the trick for the 3 by 5 calendar pattern below?

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Teacher reflection:

The students may need help on creating their own puzzle because they usually have learned how to solve the math problems, but not learned how to come up with their own problems. You may help them by giving them some suggestions like: “you can choose days that form a triangle.” Give extra credit if someone creates a great puzzle to motivate them. Depending on student level, you can make it easier or harder by decreasing or increasing the number of days. You can work more on the board to accommodate students who are more visual. Giving a small quiz about the calendar puzzle a week later would reinforce their understanding or at least let you know whether the lesson was effective.
I did the activity with the kids in Presidio. At first, they thought I used some magic or I were cheating. After I explained to them that I just used what they have learned and showed it to them, the kids were very interested. Some of them came up with different ways to show the puzzle, and I let them explained their ways with the class. The kids totally engaged with the activity even though they had some troubles with creating their own puzzles.

REFERENCES:


http://math.rice.edu/~lanius/Lessons/index.html