Rep-Tiles
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Introduction & Background

In this activity, we want students to discover and explore algebraic expressions through a special kind of tiling of the plane. This activity is geared towards sixth graders (advanced) through 10th graders in high school. Rep-tiles are geometric figures such that \( n \) copies can fit together to form a larger, similar figure. The students experiment with various shapes and values of \( n \) through this activity. Spatial sense is encouraged by the need to visualize and perform transformations with the shapes involved. This lesson was adapted from [1]. It originally appeared in the January-February 1996 edition of Mathematics Teaching in the Middle School by Linda Fosnaugh and Marvin Harrell.

A tiling is a partitioning of the plane into regions, or tiles. It is commonly known that any triangle, quadrilateral, and regular hexagon will tile the plane. A rep-tile tiles the plane. Another way that one can think of a rep-tile is as a puzzle piece, where a larger similar figure is the entire puzzle. For example, four congruent squares fit together to form another square. The smaller square, the puzzle piece, is a rep-tile. In addition, four copies of the large square can be fitted together to form an even larger square. By repeating this process infinitely many times with still larger squares, we can tile the plane. Other examples include the triangle and the parallelogram (see figure 1). Four congruent triangles or squares or parallelograms create a similar figure; thus we call this tile a rep-4 tile. It should be noted that a rep-\( n \)-tile exists for any natural number \( n > 1 \). In other words, for any natural number \( n > 1 \), a tile exists in which \( n \) copies can be fitted together to create a larger similar figure.

Through this activity, students will come up with their own rep-tiles and finally represent the area or perimeter of their rep-tiles in an algebraic expression using the algebra tiles provided. Rep-Tiles not only present an alternative way to tile the plane and learn more about algebra, it also help students acquire spatial sense.

Learning Objectives

- describe, classify, and understand relationships among types of two-dimensional objects using their defining properties
- examine the similarity of objects using transformations
- identify geometric and number patterns
- create and extend patterns
- practice problem solving
- identify and simplify algebraic expressions
Materials

- Rep-Tiles worksheet
- pencils
- graph papers
- construction papers
- rulers
- scissors
- algebra tiles
- pattern blocks or colored transparency sheets (optional)

Instructional Plan

This is primarily an independent exploration activity with minimal lecturing. The instructor mainly guides the activity with a discussion in the beginning and explains some of the concepts in the middle and conclude with student presentations of what they learned and the examples they come up with at the end of the activity.

This activity is ideal for middle school students, although some high schoolers may benefit. It may take one to two periods to completely cover the whole lesson depending on the students’ focus and background.

1. Divide the classroom into groups of 4-5 and begin the lesson by asking students where they have seen examples of tiling. Let them give some examples to the class and add some of these common uses of tilings if not mentioned: covering of walls, floors, ceilings, streets, sidewalks, and patios.

2. Introduce or review (depending on grade level and prior knowledge) such concepts as tilings (tessellations), congruent figures, and similar figures. The teacher may also want to review basic shapes such as squares, equilateral triangles and parallelograms.

3. Introduce rep-tiles using squares, equilateral triangles, and parallelograms by using an over-head or document reader (elmo) to illustrate examples of rep-tiles. Then have the students guess and discuss what they think a rep-tile is. Once they have made some good guesses with guidance, show a non-example of a rep-tile. Give students a chance to redefine what their guesses of a rep-tile are. Figure 1 below are some examples of rep-tiles and figure 2 are non-examples.
Figure 1: a. A rep-4 tile made from four congruent triangles creating a similar figure. b. Another rep-4 tile made from four parallelograms. c. Rep-4 tile made from four squares.

Figure 2: Non-examples of rep-tiles. The geometric figures are not made up of copies of a smaller similar figures.

4. Finally, after showing the above examples, distribute the Rep-Tiles worksheets without going into questions they might have. One of the goals of the activity is independent exploration, so we want them to answer their own questions through the activity.

5. After students have worked through the activity sheet, each group presents its findings and algebraic expressions to the rest of the class. Following the presentations, a class discussion summarizing their findings is beneficial. Students should realize that any of the similar figures, no matter their size, can be used to tile the plane.
Rep-Tiles Worksheet

1. Draw a triangle on your graph paper.

   Using construction paper, cut out four copies of this triangle. Will your triangle fit together to form a larger triangle?

   Is the larger triangle similar to the smaller triangles? Measure the angles and sides of both the larger triangle and one of the smaller triangles to make sure the triangles are similar.

   Is your triangle a rep-tile? Explain your answer.

2. Cut out four copies of the larger triangle constructed in number 1. As in number 1, see if the larger triangle is a rep-tile. How many triangles were needed?

   Were the same number of triangles needed in 1 and 2?

3. Repeat 1 and 2 above with two other triangles.

4. Draw a parallelogram. Make four copies of the parallelogram and see if it is a rep-tile. Discuss why the parallelogram is or is not a rep-tile.

5. Draw a 1x1 square in your graph paper.
   Use this square to extend this pattern to form the next four rep-tiles.
   Describe all the patterns you have found in terms of the area of the square, the perimeter, and the difference between one square from the previous and the next one.

6. Use construction paper to create four copies of each of the following figures. Which figure(s) is a rep-tile and which is or are not? Explain how you knew.

   ![Diagram](image)

7. Each rep-4 tile you found in numbers 1 through 4 is also a rep-9 tile. That is, nine copies of any of these figures can be used to create a larger, similar figure. Show that each of your reptiles (from 1 to 4) is also a rep-9 tile.

8. Draw an isosceles right triangle and show that it is a rep-2 tile.
9. Show that a 30–60–90 degree triangle is a **rep-3 tile**.

10. Show that a right triangle whose legs measure 2cm and 4cm is a **rep-5 tile**.

11. Pick two of the **rep-tiles** you created above. Use the algebra tiles to create two algebraic expressions for the perimeter of each rep-tiles and two algebraic expressions for the area of each rep-tiles.

12. What polygons did you use to create your rep-tiles?

13. What geometric concept can rep-tiles be compared to? What’s the difference?

**Extensions**

14. Each of the polygons in 1 – 9 can be divided into smaller shapes that are congruent to each other and similar to the original polygon. For example, a **rep-4 tile** can be subdivided into four smaller congruent copies that are similar to the original tile. Subdivide six of the polygons in 1 – 9 in such a way. (Hint: Think about how you put copies of a triangle together to form a larger similar triangle.)

15. Students may wish to experiment with creating tilings on the computer. The figures below provide examples of computer-generated tilings.
SOLUTIONS: Rep-Tiles Worksheet

1. Draw a triangle on your graph paper.

Using construction paper, cut out four copies of this triangle. Will your triangle fit together to form a larger triangle? yes.

Is the larger triangle similar to the smaller triangles? yes. Measure the angles and sides of both the larger triangle and one of the smaller triangles to make sure the triangles are similar.

Is your triangle a rep-tile? Explain your answer. Yes, because it tiles the plane.

2. Cut out four copies of the larger triangle constructed in number 1. As in number 1, see if the larger triangle is a rep-tile. How many triangles were needed? four.

Were the same number of triangles needed in 1 and 2? yes.

3. Repeat 1 and 2 above with two other triangles.

4. Draw a parallelogram. Make four copies of the parallelogram and see if it is a rep-tile. Discuss why the parallelogram is or is not a rep-tile.

5. Draw a 1x1 square in your graph paper.
   Use this square to extend this pattern to form the next four rep-tiles.
   Describe all the patterns you have found in terms of the area of the square, the perimeter, and the difference between one square from the previous and the next one.

6. Use construction paper to create four copies of each of the following figures. Which figure(s) is a rep-tile and which is or are not? Explain how you knew. They are all rep-tiles because you can tile the plane with each of them.

7. Each rep-4 tile you found in numbers 1 through 4 is also a rep-9 tile. That is, nine copies of any of these figures can be used to create a larger, similar figure. Show that each of your reptiles (from 1 to 4) is also a rep-9 tile.
8. Draw an isosceles right triangle and show that it is a rep-2 tile.

9. Show that a 30–60–90 degree triangle is a rep-3 tile.

10. Show that a right triangle whose legs measure 2cm and 4cm is a rep-5 tile.

11. Pick two of the rep-tiles you created above. Use the algebra tiles to create two algebraic expressions for the perimeter of each rep-tiles and two algebraic expressions for the area of each rep-tiles.

12. What polygons did you use to create your rep-tiles?

13. What geometric concept can rep-tiles be compared to? Tessellation. What’s the difference? Tessellation is the study of self-replicating tiling pattern in which several congruent tiles may be joined together to form a larger tile that is similar to the original. A rep-tile is a polygon that can be dissected into $n$ smaller copies of itself.
Assessment Options

1. Students may be assessed by their responses to number 1 – 9 of the worksheet.

2. For an individual project, students can pick their favorite rep-tile and create a poster illustrating a tiling of the plane. Some of the more interesting posters are those in which students color their tile as a reptile. This project not only reinforces the notion of rep-tiles but also aids the students in making a connection between mathematics and certain principles of art.

Conclusion

With the introduction of rep-tiles in a classroom setting, students can visualize and represent geometric figures with special attention to developing spatial sense as well as explore transformation of geometric figures. Creating a tiling pattern requires such mental imagery as visualizing the possible rotations and placements of a tile in the tiling pattern, thus further developing spatial sense. Moreover, students get to see the connection between geometry and algebra through the algebraic expressions they create from the algebra tiles. In addition, the construction of a tiling with rep-tiles allows students to see how congruent copies of a polygon can be fitted together to form a larger similar copy of the polygon. After discovering the pattern, the students may see that each tile can be dissected into smaller, similar copies of itself, thus discovering how any of the similar figures can be used to generate an entire tiling.

Activity Reflection

References