

Pick's Theorem Suggestions for Instructors

#1 This is fairly challenging at first. So when trying this you may want to suggest students

- consider diagonals with one unit horizontal for several vertical.
- Consider whether a triangle can have 4 boundary points
- I have provided a few examples to maybe encourage students to work toward

#2 I will assign a polygons off the set of 10 on page 3. Some may need some creativity to find area of. While doing this consider:

- breaking complex polygons in to several triangles and adding them up
- enclosing polygons in a shape you know how to find the area of (a square or rectangle for example) and subtract any extra triangles outside your original polygon.

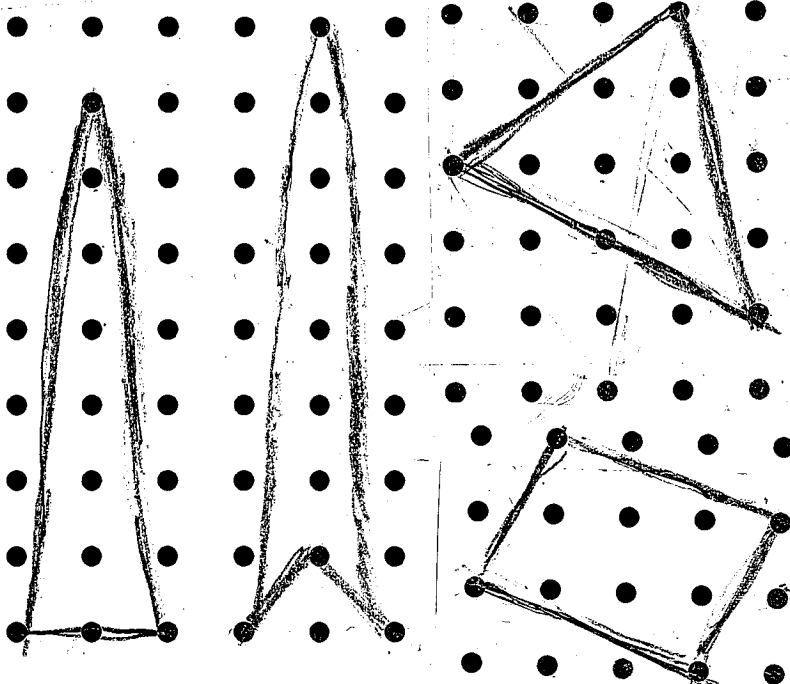
#4 The actual formula is $i+b/2-1=A$. To assist students in seeing this consider the following:

- some areas have halves how might one get halves
- also notice the ones with halves left over have an odd number of boundary points
- hopefully this will help students guess to divide b by 2. from there adding I seems natural, but the subtraction of one may seem unusual. Experience at Missions is they don't consider constants as part of relating data. But it is fairly see to observe that -1 needs to be there.

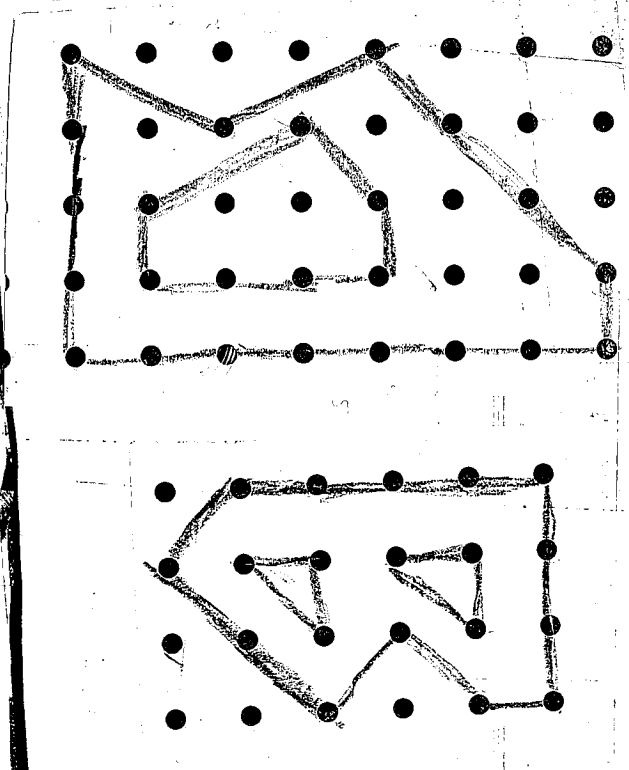
#5 the formula here is $i+b/2+h-1$. I have included examples of polygons with holes.

#6 This problem was included originally because the next session was going to be Farey fractions. However, just the same one can see the Farey fractions in the polygon defined in #6 by dividing the y value by x value in the visible points

Polygons with 4 boundary points and 6 interior points

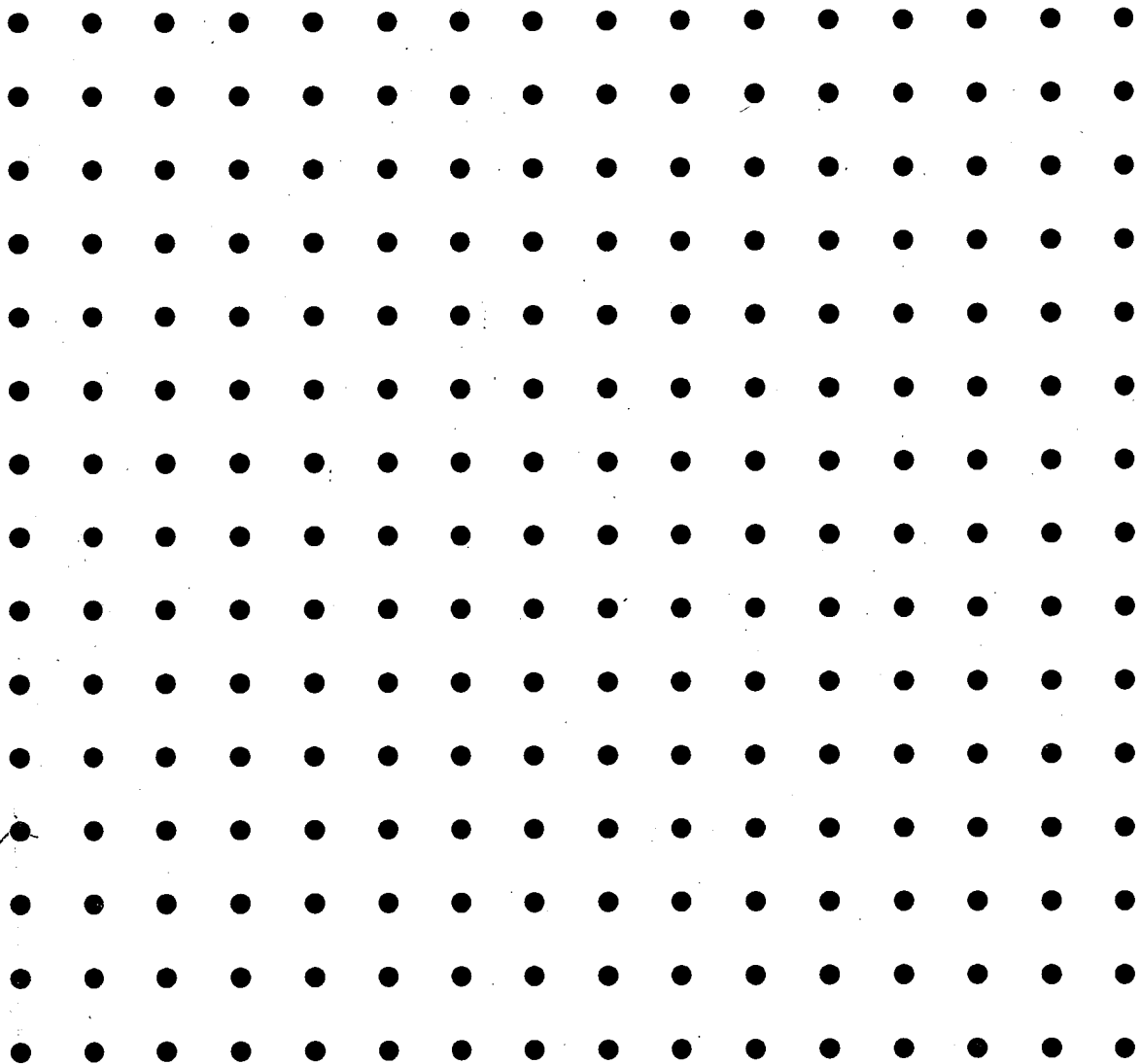


Polygons with holes



Pick's Theorem
SFMC 9/21/2009

1. Have each person in the group make a simple polygon (no holes) with 4 boundary points and 6 interior points. Be creative and try to make a polygon different from your neighbor's. Remember the polygons do not need to be convex. Compute the area for each polygon. What do you notice?



2. Look at the sheet of polygons and find the interior points, boundary points, and area for the polygons your group was assigned.

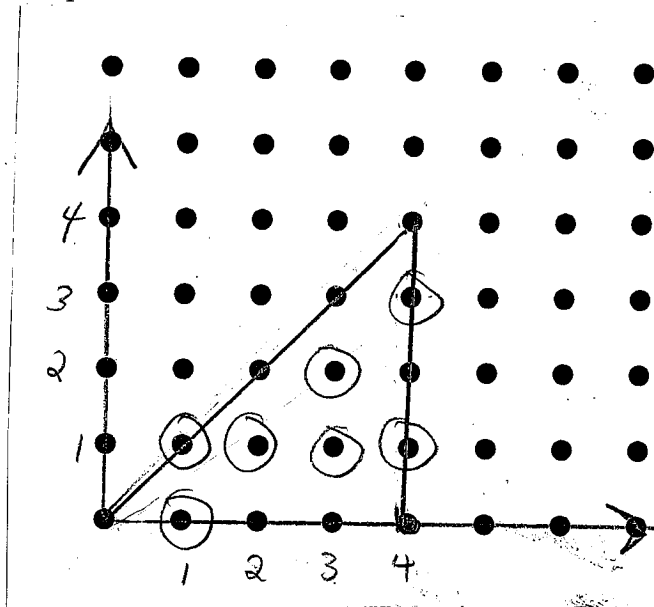
3. Fill in the following chart with the information from your polygons as well as the polygons from other groups.

Interior Points	Boundary Points	Area

4. Try to find a way to numerically relate the interior points, boundary points, and area from the above data. Does this relation explain what happened in #1?
5. Notice all the polygons so far have no holes. There is a way to relate interior points, boundary points, area, and holes. Try drawing some examples on the sheet without polygons and making another chart with interior points, boundary points, number of holes, and area.
6. The integer point (x,y) is considered **visible** if the line segment from the origin to the point (x,y) passes through no other points. For a given n , an integer greater than zero, draw the triangle with vertices $(0,0)$, $(0,n)$, and (n,n) . Mark all visible points in the triangle. What do you observe about this polygon and how does it relate to the fractions between 0 and 1 with denominator at most n ?
For example, the following is the picture for $n=4$.

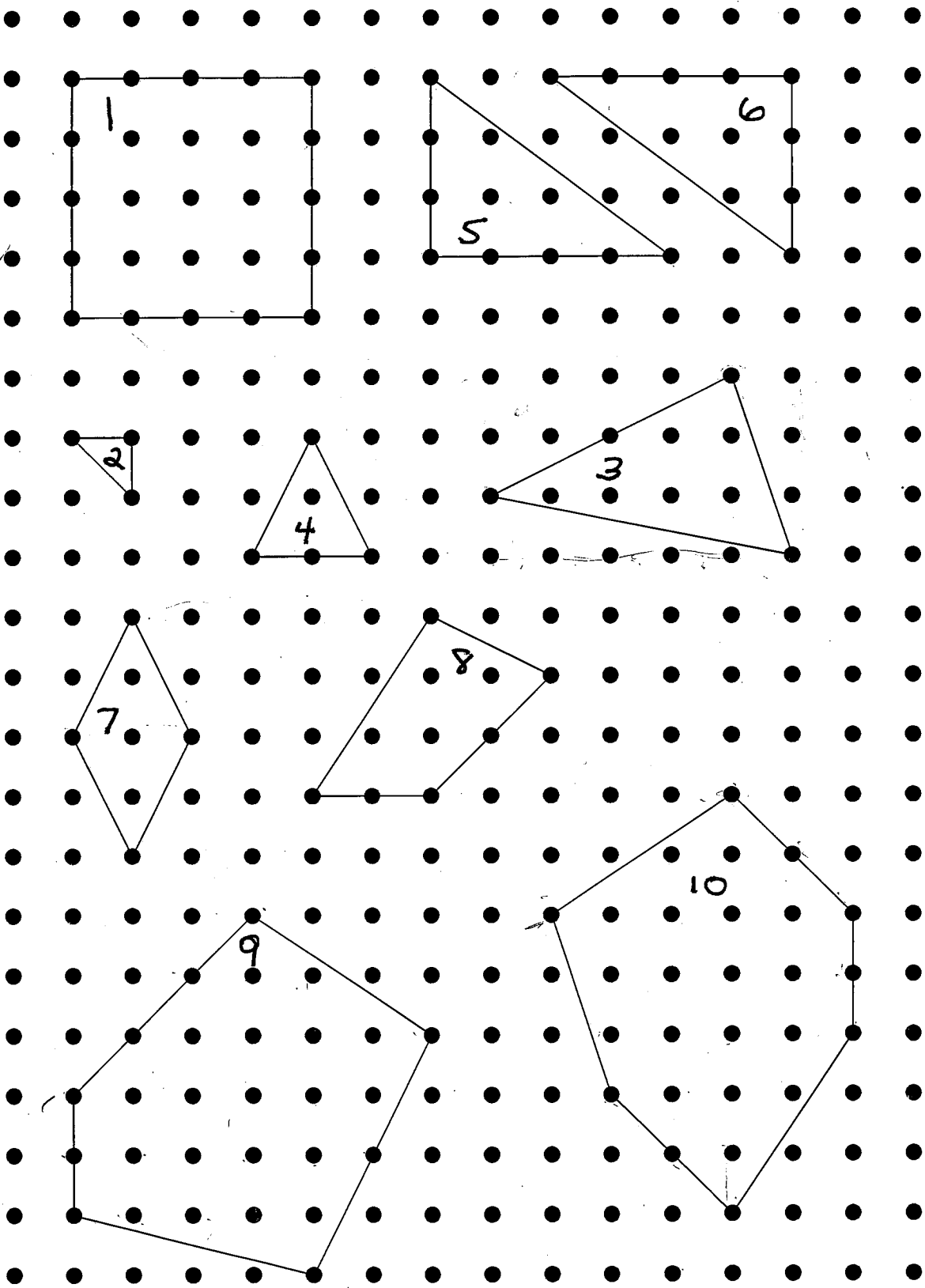
$$F_5 = \left\{ \frac{0}{4}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, 1 \right\}$$

Try some of your own!

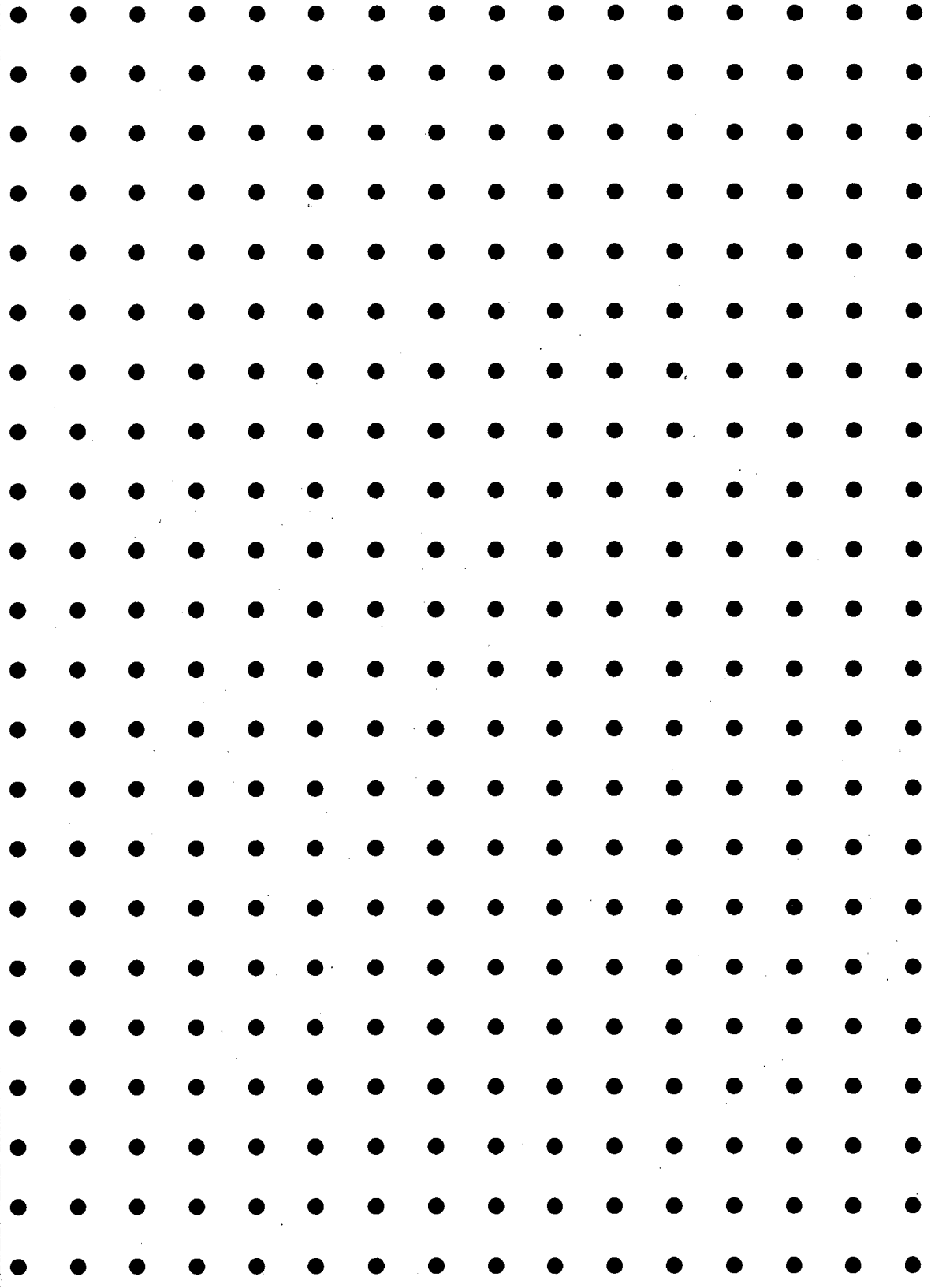


$$3+4-1=6$$

5 7 ✓
1 8 ✓
4 10 ✓
2 9 ✓
6 3



6
3+2-1



7