1 The Background

This lesson was done at Thurgood Marshal High School on the 21st of February with Mr. Pablo’s freshman algebra classes. Rika and I began the brainstorming with both Ms. Lyon and Mr. Pablo well into the last semester about possible topics that the students could use help with. The idea that students and most people for that matter have issues with fractions was highlighted as one possible topic.

After brainstorming and developing more I thought that the best possible way of showing this idea was to show equivalence relations between various fractions. Within the area of abstract algebra we call fractions a ”localization”, now this is very a technical term but the just of it is that you want to create fractions out of whole numbers and the only way of doing that is to create all possible fractions and then define what is equal. We as mathematicians call this modding out by an equivalence relation. Rika had the idea that we could do this via Cuisenaire rods. Until that moment I had no idea what these were let alone how to spell them. But after further inspection I thought she was correct and we went about with the lesson development.

The main idea was to keep the lesson as ”freeform and fun” as possible, to allow for discovery and relationships that they come up on their own and then be able to share them with the groups and the class and then allow us to explain or help explain what it is they are seeing.

2 The Lesson Plan

2.1 Intro

Begin the lesson by introducing the students to the idea that fractions are nice despite their tendency to shy away from them. Put up a LONG decimal such as:

\[
5.053097345132743
\]

and ask the students what would they do if they were asked to add another number to this? or they had to multiply this number to another one? most of them will say use a calculator or to truncate the decimal. But what if they were using this number to send a rocket to the moon? or build a bridge? Would they want the engineers to just drop a bunch of numbers? NO! (even though they do!)

Well with a little inspection we actually see that this number is in fact:

\[
571/113
\]

Now which looks prettier?
Ok so today we are going to talk about relationships of numbers but hidden in this is actually fractions!

2.2 The Activity

Pass out, two sets of colored rods to each of the tables. The students should have paper and work together as a group and it doesn’t matter if different tables have two different colors but JUST two and no more. Now ask the students to put a line together of just one color and then try to match the other color in a line of the same height.

So for example: Say table 1 was given purple rods and yellow rods. Maria at the table makes a line of the purple rods and Robbie uses the yellow rods and makes a line next to it so that the lengths are equal. They then count them and find that,

\[ \frac{5}{p} = \frac{3}{y} \]

This is one relations! Now have the students write down TWO other relations with the same numbers, i.e.

\[ \frac{p}{3} = \frac{y}{5} \]

and

\[ \frac{y}{5} = \frac{3}{3} \]

Have the students continue to do this with different colors (BUT every line of blocks must be the same color no mixing!) and writing down the relations that they see and have them manipulate them to solve for each color as above.

Have them try making 3 equal lines of different colors as well! What do they notice about the numbers they are seeing?

Finally, have the students try two color rods that are ALMOST the same length. How many of each rod do they need? Do they notice a pattern with what number the colors are supposed to represent? (Hint: common multiples!)

So what is going on? In "lower" mathematics we are just taught that \( \frac{2}{4} = \frac{4}{8} = \frac{1}{2} = \cdots \), but why? Inherently having 2 of 4 things is different than having 1 of 2, etc. and this is the reason that a lot of people have an issue with these concepts, not to mention that fractions are not taught as "division".

So how do we think about these things? Well we can say that two fractions are equal if:

\[ \frac{n_1}{d_1} = \frac{n_2}{d_2} \iff d_2 \times n_1 = n_2 \times d_1 \]

This is how we find define equal fractions, especially if there isn’t a number that can divide \( d_1 \) or \( d_2 \).

2.3 Extension

Now back to the fraction bars. What did you notice when the rods where very close in length? The students should notice that they get reciprocals of the represented length of each, i.e suppose we are using orange = 7 units and green = 6 units, then we will need 7 green and 6 orange to make an equal length, but why? Have the students play around with this and test their intuitions!
3 The Results

The day we tried this activity Mr. Pablo was out ill so there were MANY behavior issues for many reasons so the lesson didn’t go out off without a hitch even though some students got the fact that they needed to make a least common multiple to get the equal lengths! However I am going to be doing the lesson again with Ann’s class coming up soon and I feel like it will go MUCH better!