1 Introduction

Tessellations also known as tilings are a collection of polygons that fill the plane with no overlaps or gaps. There are regular tessellations that tessellates with just one polygon and semi-regular tessellations that use two or more regular polygons. Tessellations are a good way to introduce students to the beauty of mathematics which can be very artistic and interesting to view and study.

There are some amazing tessellation tilings that are wonderful works of art. A lot of the artwork by the famous graphic artist M.C. Escher (1898-1972), is just mind blowing and just fun to look at. Tiling was a favorite means of expression for the artist who repeatedly expressed his love for this art form.

Ecsher, M.C., Fish. 1964
2 Learning Objectives

My lesson involves using polygonal relationships to form tessellations. Our goals are to understand and create tessellations and tilings, to learn concepts and notations important for understanding and discussing polyhedra. Also which types of polygons can we use in order to make a regular tessellation and what are the conditions are needed for irregular tessellations. The lesson involves Zome tools which should be fun and hands-on for the students. The main objectives for the students to take away from this lesson are to:

• Recognize and explore the properties of tessellations.
• Identify and examine symmetry in geometric figures.
• Describe, and classify polygons, examine the role of mathematics in society and nature.

3 Materials required

Class Set of Zone tools.
Challenging Tessellation Pictures

Time required 35-50 mins

4 Mathematical Background

Definitions:

1. **Polygons** are 2-dimensional shapes. They are made of straight lines, and the shape is "closed" (all the lines connect up).

2. **Regular Polygons** are polygons that are equiangular (all angles are equal in measure) and equilateral (all sides have the same length).
3. Symmetry is the property that a figure coincides with itself under an isometry, where an isometry is an action that preserves size and shape. There are three main types of isometries, reflectional, rotational and translational.

**Types of Symmetry:**

(a) **Reflectional symmetry.** An object has reflectional symmetry if you can reflect it in a way such that the resulting image coincides with the original. Hold a mirror up to it, its reflection looks identical.

(c) **Rotational symmetry.** An object has rotational symmetry if it can be rotated about a point in such a way that its rotated image coincides with the original figure before turning a full $360^\circ$.

(d) **Translational symmetry.** An object has translational symmetry if you move it along a straight path without turning it.

4. A **Tessellation** is a repeated geometric design that covers a plane without gaps or overlaps.

5. A **Regular Tessellation** is a highly symmetric tessellation made up of congruent regular polygons. Only three regular tessellations exist: those made up of equilateral triangles, squares, or hexagons.

6. A **semi-regular** tessellation uses a variety of regular polygons; there are eight of these.

There are only three regular polygons that can be used to tessellate the plane to form a regular tessellation, the triangle, square, and hexagon.
The reason for this is the angle measures of these polygons. The angle around each vertex in a tessellation has to be 360 degrees. Therefore the interior angle degree of a regular polygon should divide into 360 evenly. The interior angles of the triangle, square and hexagon are 60, 90, and 120, respectively which works. There does not exist any other regular polygon with this property.

6 Lesson Overview

1. The aim is to get students working together in groups using a discovery method for students to learn the ideas of polygons angles and construction using the Zome System. For students who are new to the Zome tools introductory about the Zome System may be helpful to get students comfortable with using the tools. 5-10 mins

2. Once the introduction is over, students will be broken up into groups of four or five students and handed out the worksheet for this activity. The first few questions are to get students thinking or recalling properties about tessellations such as vertex conditions, types of tilings and different types of symmetries tilings can have. Students will also have time to get familiar with the Zome tools by making simple constructions 10-15 min

3. Then each group will be giving a challenging construction that they will have to work together on to complete. These can be assigned arbitrarily by just assigning one of the challenge questions to each group. 20-25 mins

4. If a group finishes before the end of the activity they can try to draw their own tessellations for their category and then try and see if they can use the Zome System tools to construct it. They will need to explain how they know that their constructing will span the whole plane with no gaps.
I presented this activity in a 10th grade geometry class and it actually lasted longer than I expected it to. The class is a part of the algebra project so the students have two periods of geometry each day except for two days a week where the school has a block schedule. The activity lasted for about a period and a half or roughly around 70 minutes which Ann gave them as a break right after a test they had, and a week before their finals. The students for the most part ignored filling out the worksheets I provided, none of them turned anything in, but most of the students attempted to build some of the tessellations listed and pictured on the worksheets.
The lesson plan was pretty open ended and designed for exploring the topic of tessellation and tilings on their own and figure out and see on their own what works and what doesn’t. The students enjoyed working with the zome tools and most of them did participate and attempt to build the tessellations. I have provided a few photos listed below. Ann suggested that we do the activity again since now the students have had an introduction to zome tools, and so they can be more focused on completing the worksheets. I suggest if zome tools are used with this activity, make sure students have some experience playing around with the pieces to speed the process up.
Patterns in Tessellations Worksheet

1. Think of 3 different objects in the world around you that have tessellating patterns
   ○ Item 1:
   ○ Item 2:
   ○ Item 3

2. Try drawing each of these patterns

3. Which types regular polygons can you use to make a tessellation
   ○ Item 1 polygon:
   ○ Item 2 polygon:
   ○ Item 3 polygon:

4. Which types of symmetry does each tessellation use?
   ○ Item 1 uses __________________________
   ○ Item 2 uses __________________________
   ○ Item 3 uses __________________________

5. How many semi-regular tessellations are possible?
Group Challenge Questions

1. **Non convex Tiles.** Make a Zome tessellation with non-convex quadrilateral and list the types of symmetry present in the tessellation.

2. **Pentagon Tiles.** Make a Zome tessellation that uses regular pentagons and just one other polygon and list the types of symmetry present in the tessellation.

3. **Kepler’s Tessellations.** The German astronomer Johannes Kelper who discovered the planets have elliptical orbits, was also interested in the problem of tessellations that involve pentagons. The figures replicate some patterns he published involving regular pentagons, regular decagons, and other polygons. Make one of these with the Zome System and list the types of symmetry present in the tessellation.
4. **Pentagons and Triangles.** The figure below shows a pattern with regular pentagons and isosceles triangles. Each pentagon touches six surrounding pentagons. Make this pattern with the Zome system and list the types of symmetry present in the tessellation.

![Pentagon and triangle tessellation](image)

5. **Spiral Tessellations.** The figures below show some interesting spiral tessellations. In the first, there is one regular pentagon surrounded by identical irregular equilateral hexagons. Explain how to extend it to infinity. The second is a double spiral composed of isosceles triangles. If you slide the bottom half to the left by the length of the side of the triangle, you would have a pattern with ten-fold symmetry. The third is a one-arm spiral, using a concave equilateral pentagon. Construct each of these with zome system and list the types of symmetry present in the tessellation.

![Three spiral tessellations](image)
Extra Questions

1. Draw your own tessellations in your category and then try and see if you use the Zome System tools to construct it.

2. Can you build a 2 dimensional model which has reflection symmetry but no rotational symmetry? Try to build the most intricate and beautiful model you can.

3. How many things can you name which have reflection symmetry but no rotational symmetry?
Bibliography


