Math 728
Lesson Plan

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Topic: Probability involving sampling without replacement and dependent trials.

Grade Level: 8 - 12

Objective: Compute the probability of winning in several blackjack situations. Observe the difference between the theoretical probability and relative frequency.

Time: 50 minutes

Materials: Several standard decks of cards, calculator

1 Introduction

In Double Down, three students are tied up in a money-laundering scheme involving money won in the card game blackjack. In this activity, we discuss a simplified version of blackjack and analyze the probability of winning in various situations. Because cards are drawn from the deck and not replaced, the probabilities are based on the concepts of sampling without replacement and dependent trials. (Note: aspects of betting are not considered in this activity.)

1.1 Discuss with Students

Blackjack is essentially a card game between two people, the player and the dealer. (There may be other players at the table, but each player is only playing against the dealer.) To win, a players hand must have a value closer to 21 than the dealers hand, without going over 21.

Before starting this activity, you will want to review the rules of blackjack with your students. Although some students may be familiar with the game, there are different variations of the game, and it is important that all students understand the rules used in this activity.

Each player is dealt two cards, and can take as many additional cards as he or she wishes. The dealer's hand must be played according to certain rules, with no choices involved. Aces are worth either 1 or 11 (at the player's discretion); tens, jacks, queens, and kings each worth 10; and all other cards are worth their face value. (Note that the suits of the cards do not matter in blackjack.) The term blackjack means that you get a value of 21 with only two cards an ace worth 11 and a 10, J,

Q, or K worth 10.

1.1.1 Simplified Rules of Blackjack

1. There is a dealer and from one to seven players at a table.

2. A deck of cards is shuffled and cut (several decks of cards may be used). The dealer deals two cards to each player and two cards to himself one face up and one face down.

3. Each player in turn may request one or more additional cards, with the goal of attaining a total value as close to 21 as possible without going over. A player immediately loses if he or she goes over 21.

4. When all players are satisfied with their current totals, the dealer may take additional cards according to the following rules: the dealer must hit (take another card) if the current total is 16 or less, and the dealer must stand (not take another card) if the current total is 17 or more.

5. Any player with a total greater than the dealers total wins. If the dealer goes over 21, all players with a total of 21 or less win.

1.1.2 Probability and Dependent Events

In the activity, students will be asked to find the probability of being dealt a black-jack with the first two cards of a deck. You could review the needed probability concepts by finding the probability of being dealt two aces in a row.

$$\text{Probability} = \frac{\text{number of successes}}{\text{number of possible outcomes}}.$$

All of the outcomes are equally likely in this activity, so to find the probability of two dependent events, multiply the probability of the first event and the probability of the second event given the first event.

Example 1. Probability of drawing an Ace from the standard deck of cards is $P(\text{ace}) = \frac{4}{52}$, since there are 52 cards and 4 of them are Aces.

When a card is dealt, it is not replaced in the deck - there are now 3 Aces left in the remaining 51 cards. So $P(\text{ace, after ace is drawn}) = \frac{3}{51}$.

Multiply to find the probability of being dealt two aces in a row: $P(\text{ace, given ace is drawn}) = \frac{4}{52} \frac{3}{51} = \frac{12}{2652} = \frac{1}{221} \approx 0.0045$

1.1.3 Answers to hand-out questions:

1. answers vary but will be around $\frac{2}{50}$ or $\frac{3}{50}$

2. answers

   (a) $52 \times 51 = 2652$
   (b) $2 \times 4 \times 16 = 128$
   (c) $\frac{2 \times 4 \times 16}{52 \times 51} \approx 0.0483$

3. answers

   (a) answers vary but will be around $\frac{2}{50}$ or $\frac{3}{50}$
   (b) $\dfrac{2 \times 8 \times 32}{104 \times 103} = 0.0478$; $\frac{2 \times 12 \times 48}{156 \times 155} \approx 0.0476$
   (c) The probability is slightly less for 2 and 3 decks, but the probabilities are close to each other. Students might have expected to see a greater difference in the probabilities when more decks of cards are used.

4. If the dealer shows a 7, he could have a total of 18 with an ace, a total of 17 with a 10, J, Q, or K, or a total of 16 with a 9. If he has less than 16, he could get additional cards so his total exceeds 16. With a 6 showing, the dealer could have a total of 17 with an ace, but would be forced to hit with any other card. His chances of going over 21 are much higher.

5. 0.11456

2   Student handout

In Double Down, three students are tied up in a money laundering scheme involving money won playing the card game blackjack. Blackjack is essentially a game between two people the player and the dealer. To win, a players hand must have a value closer to 21 than the dealers hand, without going over 21. The dealers hand must be played according to certain rules, with no choices involved. You (the player) are dealt two cards, and can take as many additional cards as you wish. Aces are worth either 1 or 11 (at your discretion), tens, jacks, queens, and kings are each worth 10, and all other cards are worth their face values. (The suits of the cards do not matter.)

In the problems below, assume that you are the only player and only a single deck of cards is being used (unless indicated otherwise).

2.1   Finding the Probability of Being Dealt a Blackjack.

The term blackjack means that you get a value of 21 with only two cards (an ace and a card with that is worth 10). What is the probability of being dealt a blackjack with the first two cards of a single deck of cards?

1. First, explore the experimental probability of being dealt a blackjack with the first two cards.

   (a) Simulate this problem with a deck of cards. Shuffle the cards and deal the top two cards from the deck. Check to see if they form a blackjack. Replace the cards in the deck and shuffle. Again, deal the top two cards and check to see if they form a blackjack. Repeat this 50 times. Then find the experimental probability of being dealt a blackjack with the first two cards of a single deck.

2. Now find the theoretical probability of being dealt a blackjack with the first two cards.

   (a) When you are dealt two cards, the order of the cards matters because only one card faces up for example, the hands Q, 8 and 8, Q are different. How many different pairs of cards can you be dealt? (Hint: Think of the number choices for the first and second cards.)

   (b) If a pair is a blackjack, then you were either dealt an ace followed by a 10, J, Q, or K, or a 10, J, Q, or K followed by an ace. How many different

blackjack pairs are there? (Remember, there are four cards of each of the 13 denominations in the deck.)

(c) What is the theoretical probability of being dealt a blackjack?

3. In some blackjack games, several decks of cards are used. Do you think the probability of being dealt a blackjack on the first two cards will increase, decrease, or stay the same if more than one deck of cards is used? Answer the following questions to find out.

   (a) Combine 2 decks of cards. Again, perform 50 trials in each case. What is the experimental probability?

   (b) Calculate the theoretical probability of being dealt a blackjack using both two and three decks of cards.

   (c) Were the probabilities of being dealt a blackjack from 1, 2, and 3 decks of cards what you expected?

2.2   Basic Blackjack Strategy and Probability

In the actual game of blackjack, you must make your decision whether to hit or stand after seeing only one of the dealers two cards (the other card is face-down on the table). If your total is high (17 or more) you would want to stand, and if your total is low (11 or less) you would want to hit.

The difficult decisions arise when your total is between 12 and 16. Many books and Internet sites publish tables indicating the better strategy in all cases.

4. Look at the table below.

Player Total	Dealer Up Card	
	6	7
16	Stand	Hit

Give an intuitive reason why a player with a total of 16 should stand if the dealer is showing a 6 but hit if the dealer is showing a 7.

5. Challenge
Suppose you start another hand with 1 fresh deck of cards. Your first two cards total 16 without using an ace, and you are reasonably sure that the dealers two cards total 20.

Your cards   **Dealer's cards**
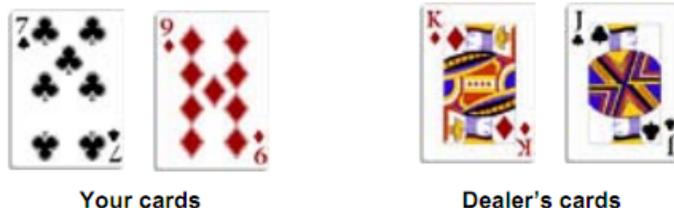
What is the probability you will win this hand by hitting and obtaining one or more cards worth a total value of 5?

Hint: There are 15 different ways to draw cards with a total value of 5. Find the probability for each of these card combinations, and add the probabilities. Remember that four cards have already been dealt, so there are 48 cards remaining in the deck. Also remember that there are four cards of each denomination.

3 Reflections

After conducting this lesson in my AP Statistics course there are few things that I have become aware of. Many students are not aware of the rules of the game, so in order for us to fully gain benefit of this lesson, it might be wise to have 10-15 minutes play session in the meetings before, so students are more aware of the rules. We can also ask them to record their findings (i.e., how many blackjacks did they get in the number of plays they made). This will make the pace of the activity better and let students focus on probability.

Recently I've heard a big debate about introducing Blackjack to the middle school students. Many parents and teachers found that this game sparks the interest in mathematics and finding a best possible strategy to solve problems. With this said, the activity I proposed can be separated in the whole series of small sessions (may be a warm up, or end of the class activities) in 6-12 grade.

References

[1] Tom Butts, www.education.ti.com/go/NUMB3RS, UT Dallas.

[2] The Strategies of Blackjack, http://www.bjmath.com/index.html.