Lesson Plan: Mountain Building

Michael Henley

Introduction

Recognizing patterns is an essential skill for a mathematician and patterns appear in many different places in the “real” world as well. It is important for students to be able to recognize patterns in mathematics because it leads students to a deeper understanding of the math. Recognizing patterns allows students to move beyond memorization and into understanding.

Goals

• Organize information to see patterns
• See how recursion can be used to solve problems

Materials Required

• Graph paper for each student (ruled paper will do in a pinch)
• Pencils
• Overhead projector/Chalk board/Dry erase board

Instructional Plan

This activity is primarily a student exploration of the topic with minimal teacher guidance. This activity is best done in groups.

1. Divide the students into groups of 3-4. Groups smaller than 3 may not be able to complete the activity and groups larger than 4 will be too big to effectively collaborate.

2. Explain the rules for building mountains and show the students examples of mountains drawn with 14 strokes. The first rule for building mountains is that they must start and finish on the horizon. The second rule is that they must not go below the horizon. In order to draw a mountain first choose a horizontal line to be the horizon then begin making diagonal strokes up and down until you have finished making your mountain (the boxes on graph paper come in handy here). Note that the mountain can touch the horizon and rise up again as long as it finishes on the horizon in the end.

3. Once students are clear on the rules for drawing mountains they are ready to begin. Pass out the task cards to each group and have them try and draw all the possible mountains that have 2, 4 and 6 strokes. There is one mountain with 2 strokes, 2 mountains with 4 strokes and 5 mountains with 6 strokes. Encourage the students to keep at it until they have found all the mountains.

4. After a group has found all of the mountains they need to organize them and look for patterns. This might require some teacher guidance. Some pocket questions to have ready are: How can
you use 2 stroke mountains to build a 4 stroke or 6 stroke mountain? Are any of the 6 stroke mountains a combination of smaller mountains?

5. From here students are ready to try and find all of the 8 stroke mountains (there are 14). Encourage them to articulate their strategies for building new mountains out of smaller ones.

6. After the students have found all of the 8 stroke mountains challenge them to predict how many 10 stroke mountains there are and then have them test their prediction by drawing the mountains in an organized way.

7. Conclude the lesson by having students present their strategies for building mountains to the class and any patterns that they noticed.
Mountain Building Task Card

Goal:
• Organize information to see patterns
• See how recursion can be used to solve problems

Rules For Drawing Mountains

• Begin and end on the horizon.
• Do not go below the horizon.

This is the notation we use: $M_n$ is a mountain range drawn with n strokes.

TASK 1:
Draw all $M_2$ mountains, $M_4$ mountains and $M_6$ mountains and record them on your paper in an organized way. How are you sure you’ve found them all?

QUESTIONS:
How have you organized your mountains? What patterns do you see? How else could you have organized your mountains?

CHECK POINT:
Convince your teacher that you have found all of the $M_2$, $M_4$ and $M_6$.

TASK 2:
Draw all $M_8$ mountains and record them on your paper in an organized way.

QUESTIONS:
What is your strategy for building mountains? Explain how you are sure that your strategy will find all of the mountains of a given size?

CHECK POINT:
Convince your teacher that you have found all of the $M_8$ mountains.

CHALLENGE:
Use your strategies for mountain building to predict how many $M_{10}$ mountains there are. Test your prediction by drawing them all in an organized way.
Recursion: New mountains can be made from old mountains in two ways: first by shifting the horizon down one space and adding an upstroke in the first position and a down stroke in the last; second by placing two mountains side by side.

Revisions: After teaching the lesson I found that the students had a hard time recognizing the recursion. Having them draw the $M_8$ mountains during Task 1 would give them more opportunity to see the patterns. They could then find all the $M_{10}$ or estimate how many there are for Task 2. The sets of questions after each task should be switched so that the students first focus on strategies for building mountains and then look for patterns to organize them.

Going Further: The numbers that count how many mountains can be built are called the Catalan numbers. The Catalan number $C_n$ tells how many mountains can be made with $2n$ strokes. The first Catalan number is $C_0=1$ (because we consider the empty-mountain with zero strokes) and the second Catalan number $C_1=1$. The other Catalan numbers are given recursively by the formula $C_k=C_0C_{k-1}+C_1C_{k-2}+\cdots+C_{k-1}C_0$. So for example $C_4=C_0C_3+C_1C_2+C_2C_1+C_3C_0$. The Catalan numbers appear in many different places in Mathematics such as the number of ways to cut an $n+2$ gon into triangles by cutting from vertex to vertex without cut lines crossing (so a square can be “triangulated” in two ways, a pentagon in five ways and a hexagon in 14 ways). Challenge advanced students to find the correspondence between mountains and triangulations and see if they can develop a process that will translate a mountain into a triangulation and vice versa.
Examples of mountains:

10 strokes

6 strokes

8 strokes

Building new mountains from old ones:

+ =

+ =

+ =