MATH 435 Homework 12/4–12/8

(1) Suppose R is an integral domain and M is an R-module. Prove that $\mathrm{Tor}(M)$ is a submodule of M, and that $M/\mathrm{Tor}(M)$ is torsion free. Show that $\mathrm{Tor}(M)$ need not be a submodule if R is not an integral domain.

(2) Suppose R is a ring and M and N are R-modules. Prove:

(a) If R is an integral domain, then $\mathrm{Tor}(M \oplus N) \simeq \mathrm{Tor}(M) \oplus \mathrm{Tor}(N)$

(b) If P is a submodule of M, and Q is a submodule of N, then

$$(M \oplus N)/(P \oplus Q) \simeq M/P \oplus N/Q\,.$$

Generalize.

(3) Suppose R is a ring. Prove that is M and N are free R-modules, then $M \oplus N$ is also a free R-module.

(4) Suppose R is a ring and M is an R-module. Show that a set that contains a torsion element of M cannot be a basis of M.