MATH 435 Homework 11/27–12/1

(1) Let M be an R-module, and denote the zero vector by 0_M and the zero scalar by 0_R. Show that for all $x \in M$ and all $r \in R$:

(a) $r\,0_M = 0_M$

(b) $0_R\,x = 0_M$

(c) $(-r)x = r(-x) = -(rx)$

(d) If R is a field, then $rx = 0_M$ implies $r = 0_R$ or $x = 0_M$.

(2) Let R be a ring (with 1). Prove that R is a $\mathbb{Z}$-algebra.

(3) Let R be a commutative ring. Prove that $R[[x]]$, the set of formal power series over R, forms an R-algebra.

(4) Suppose M and N are R-modules and $f : M \to N$ is a homomorphism. Prove that $\ker(f)$ is a submodule of M and that $\mathrm{Im}(f)$ is a submodule of N.