Worksheet 7: The Möbius Function

1. Show that \( \mu(n) \) is multiplicative.

2. Prove that
\[
\sum_{d|n} \mu(d) = \begin{cases} 
1 & \text{if } n = 1, \\
0 & \text{if } n > 1.
\end{cases}
\]

*Hint:* for \( n > 1 \), try induction on the number of prime factors of \( n \).

3. Prove the Möbius Inversion Formula:
\[
f(n) = \sum_{d|n} g(d) \quad \text{if and only if} \quad g(n) = \sum_{d|n} \mu(d) f\left(\frac{n}{d}\right).
\]

*Hint:* write sums like the one on the right-hand side as
\[
\sum_{d|n} \mu(d) f\left(\frac{n}{d}\right) = \sum_{de=n} \mu(d) f(e).
\]

4. Andrews 6.4.1, 6.4.3, 6.4.7, 6.4.8, 6.4.11.

5. Write down a precise statement for each definition we have given this week. For each definition, give an example and a non-example.